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A non-ordinary state-based peridynamics framework for anisotropic materials

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Highlights

- First framework for generally anisotropic materials in peridynamics.
- A damage criterion for composite materials has been employed.
- Very good agreement with numerical and experimental benchmark solutions is achieved.
- There is evidence that the horizon size may also depend on the material properties.

Abstract

Peridynamics (PD) represents a new approach for modelling fracture mechanics, where a continuum domain is modelled through particles connected via physical interactions. This formulation allows us to model crack initiation, propagation, branching and coalescence without special assumptions. Up to date, anisotropic materials were modelled in the PD framework as different isotropic materials (for instance, fibre and matrix of a composite laminate), where the stiffness of the bond depends on its orientation. In this work we propose a non-ordinary state-based formulation to model general anisotropic materials. The material properties for each particle are defined using the material constitutive matrix, rather than being defined through the bond stiffness between adjacent particles. We propose a damage criterion for composite materials to model the crack propagation behaviour for anisotropic materials. We validate the model using benchmark problems obtained with established numerical methods or experimental results. The proposed approach enables the use of general classes of material models including rocks, concrete and biomaterials.

Keywords: Peridynamics; Non-ordinary state-based; Anisotropic materials; Crack propagation

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1. Introduction

Anisotropic materials have been studied since the first works of Sih et al. [1], and ever since have attracted attention of fracture mechanics researchers. Anisotropic materials are usually brittle, almost ensuring that cracks will appear during their lifetime. The effect of cracks in an anisotropic material is more complicated than the equivalent problem with an isotropic material. For instance, the crack propagation path is also a function of the material properties, rather than depending only on the orientation of the applied load and the specimen geometry.

These materials are widely used in composites in the aerospace and automobile industries [2,3], as sensors and actuators (piezoelectric [4,5] and magnetoelectroelastic [6,7] materials) and more recently have applications in biomechanics [8,9] and even in hydraulic fracturing [10,11], just to mention some of the works. Cracks in composite materials can be responsible for complete failure of the component, resulting in economic losses or even loss of life. Damaged smart materials exhibit different electric and magnetic fields compared to the pristine material, incurring in errors of the response of a sensor for instance. Therefore, it is important to accurately quantify the effect of discontinuities in anisotropic materials.

Fracture mechanics have been studied for nearly a century, from the first work of Griffith [12] for brittle materials. Over the years a number of researchers have modelled fracture mechanics analytically [13,14] for simple problems and geometries, and more commonly using numerical frameworks, such as the extended finite element method (XFEM) [15,16] and the extended boundary element method (XBEM) [17], among many others. Nevertheless, these methods suffer when crack branching or coalescence are involved.

The phase-field method has been shown to model crack branching behaviour [18,19]. The method consists in describing the crack as an interface directly in the formulation and is used conjointly to the finite element method (FEM) [20]. Nevertheless, the method requires a fine mesh around the crack to model the interface correctly. Another drawback of the method is that it can provide unrealistic results. A novel numerical method entitled peridynamics (PD) [21] has been recently developed, and has shown great potential in fracture mechanics problems involving initiating, propagating, branching and coalescing cracks.

The peridynamics (PD) formulation is a type of non-local formulation, where the state of a point is measured over a finite distance. This framework was proposed by Silling [21], where he redefined the classical approach for continuum mechanics using an integral framework considering the forces in the bonds rather than stresses and strains as in the classical continuum mechanics. The main reason for using this approach is that the classical formulation contains partial derivatives that pose a challenge when dealing with fracture mechanics problems. The governing partial differential equations in elasticity imply that singularities will appear due to the presence of discontinuities, which is not desirable. Due to the integral form of the formulation, no special assumptions are needed to deal with singularities, such as a crack in the domain.

The first PD formulation described a continuum medium through discrete particles, interacting between each other through physical connections entitled bonds. Each bond has a stiffness associated with it, being analogous to a spring in continuum mechanics theory. However, each particle has an area of influence, interacting with all other particles within a perimeter. The radius of this perimeter is the horizon of that particle, and is a characteristic of non-local formulations (see [22,23] for other types of non-local approaches). The material properties in PD are calculated using the material parameters of the classical continuum mechanics, and also parameters from PD such as the horizon size. The tractions between different particles are in the same direction as the bond, have opposite sense and the same magnitude. This first formulation obtained by Silling was denoted bond-based PD.

The crack is formed when the bonds between particles are broken, a key feature of the PD formulation. This characteristic also enables the modelling of crack initiation without further assumptions. Additionally, crack branching can appear if elastic wave reflections generate instabilities at the crack tip, which is very difficult to model in standard numerical techniques. However this theory presented limitations with respect to the material properties. Silling has stated that the so called bond-based PD limits the Poisson's ratio of the material (1/3 for 2D plane stress and 1/4 for 2D plane strain and 3D) [21,24]. A more generalised framework called state-based PD has been developed so any material properties can be assumed without restrictions [24].

The state-based PD is divided into two main approaches: the ordinary state-based PD, which represents a generalisation of the bond-based theory, and the non-ordinary state-based PD. Some of the main differences lie in the orientation of tractions between particles and how material properties are obtained. The tractions in ordinary state-based PD are still in the same direction of the bond, but they are not constrained to have the same magnitude. A relation more similar to continuum mechanics is present due to the use of state vectors. The balance of linear and