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A marginalized unscented Kalman filter for efficient parameter estimation with applications to finite element models

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Abstract

This paper focuses on the problem of combined state/parameter estimation in dynamical systems with many degrees of freedom, for instance finite element models, using measured data from the system and nonlinear Bayesian filtering techniques for the estimation. A highly efficient nonlinear Kalman filtering technique is developed, based on a combination of linear or extended Kalman filtering for state estimation and unscented filtering for parameter learning. Such a combination is implemented by applying the principle of marginalization to the unscented transform. This method leads to a very efficient state/parameter filtering algorithm by taking advantage of the fact that Jacobians of the system equations with respect to the dynamic states, required in extended Kalman filtering, can be easily related to outputs of the finite element analysis required for forward propagation. For linear dynamical systems this approach yields an algorithm with identical accuracy as the UKF but reduced computational time, as demonstrated on several medium-size examples. For nonlinear systems, the resulting algorithm is also superior to the UKF in terms of computational time, due to the fact that Jacobians are functions of the tangent stiffness matrix of the system, whose computation is required for propagation in finite element analysis. It is also shown that, even though this algorithm relies on linearization for propagation of moments of the dynamic states, this reduction in accuracy compared to a generic UKF does not affect learning of the parameters for the systems considered herein.

Keywords: inverse problems, extended and unscented Kalman filtering, marginalization, Bayesian inference, finite element models, dynamic analysis

1. Introduction

In a world that concurrently experiences increasing availability to measured data and expanding capabilities in terms of computational modeling, an opportunity has risen for the development of non-invasive identification techniques, which are of great importance to many fields of science and engineering (e.g., monitoring of mechanical and civil structures [1, 2], non-destructive damage assessment [3], diagnosis in the medical field through imaging [4, 5] or patient-specific cardiac modeling [6]). Solving for such inverse task, i.e., learning unknown states and/or parameters characterizing a system from experimental data, should be performed in a framework that properly accounts for presence of uncertainties - measurement and modeling errors, unknown boundary conditions, random excitations and so on. With respect to parameter estimation two main approaches compete. In a frequentist approach, parameters are considered deterministic but unknown; they typically would be estimated through minimization of a loss function between the measurements and model outputs (ordinary least square for example). Uncertainties can also be accounted for by computing confidence intervals, which can be used to quantify the accuracy of the estimation procedure, but do not provide a probability density function (PDF) for the parameters ([7]).

On the other hand, the Bayesian approach treats parameters as random variables and the goal of inference is to approximate their conditional PDF knowing the noisy data $\mathcal{D}$. A prior PDF must be chosen that would

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