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Matrix-free weighted quadrature for a computationally efficient isogeometric k-method

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Abstract

The k-method is the isogeometric method based on splines (or NURBS, etc.) with maximum regularity. When implemented following the paradigms of classical finite element methods, the computational resources required by the $k-$method are prohibitive even for moderate degree. In order to address this issue, we propose a matrix-free strategy combined with weighted quadrature, which is an ad-hoc strategy to compute the integrals of the Galerkin system. Matrix-free weighted quadrature (MF-WQ) speeds up matrix operations, and, perhaps even more important, greatly reduces memory consumption. Our strategy also requires an efficient preconditioner for the linear system iterative solver. In this work we deal with an elliptic model problem, and adopt a preconditioner based on the Fast Diagonalization method, an old idea to solve Sylvester-like equations. Our numerical tests show that the isogeometric solver based on MF-WQ is faster than standard approaches (where the main cost is the matrix formation by standard Gaussian quadrature) even for low degree. But the main achievement is that, with MF-WQ, the k-method gets orders of magnitude faster by increasing the degree, given a target accuracy. Therefore, we are able to show the superiority, in terms of computational efficiency, of the high-degree k-method with respect to low-degree isogeometric discretizations. What we present here is applicable to more complex and realistic differential problems, but its effectiveness will depend on the preconditioner stage, which is as always problem-dependent. This situation is typical of modern high-order methods: the overall performance is mainly related to the quality of the preconditioner.

Keywords: Isogeometric analysis, k-method, matrix-free, weighted quadrature, preconditioner.

1. Introduction

Introduced in the seminal paper [1], isogeometric analysis is a numerical method to solve partial differential equations (PDEs). It can be seen as an extension of the standard finite element method, where the unknown solution of the PDE is approximated by the same functions that are adopted in computer-aided design for the parametrization of the PDE domain, typically splines and extensions, such as non-uniform rational B-splines (NURBS). We refer to the monograph [2] for a detailed description of this approach.

One feature that distinguishes isogeometric analysis from finite element analysis is the regularity of the basis functions. Indeed, while finite element functions are typically C^0, the global regularity of splines of degree p goes up to C^{p-1}. We denote k-method the isogeometric method where splines of highest possible regularity are employed. In this setting, higher accuracy is achieved by k-refinement, that is, raising the degree p and refining the mesh, with C^{p-1} regularity at the new inserted knots, see [1].

The k-method leads to several advantages, such as higher accuracy per degree-of-freedom and improved spectral accuracy, see for example [1, 3, 4, 5, 6, 7]. At the same time, the k-method brings significant challenges at the computational level. Indeed, using standard finite element routines, its computational

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