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A unified variational framework for the space discontinuous Galerkin method for elastic wave propagation in anisotropic and piecewise homogeneous media

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Abstract

We present a unified multidimensional variational framework for the space discontinuous Galerkin method for elastic wave propagation in anisotropic and piecewise homogeneous media. Based on an elastic wave oriented formulation and using a tensorial formalism, the proposed framework allows a better understanding of the physical meaning of the terms involved in the discontinuous Galerkin method. The unified variational framework is written for first-order velocity–stress wave equations. An uncoupled upwind numerical flux and two coupled upwind numerical fluxes using respectively the Voigt and the Reuss averages of elastic moduli are defined. Two numerical fluxes that are exact solutions of the Riemann problem on physical interfaces are also developed and analyzed in the 1D case. The implemented solvers are then applied to different elastic media, especially to polycrystalline materials that present a particular case of piecewise homogeneous media. The use of the three upwind numerical fluxes, which only solve approximately the Riemann problem at element interfaces, is investigated.

Keywords: Space discontinuous Galerkin method; Elastic wave propagation; Anisotropy; Piecewise homogeneous medium; Polycrystalline materials

Contents

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1. Introduction

Numerical modeling of elastic wave propagation is a classical problem and can be performed reliably and efficiently in a large number of cases today. However, the accurate and efficient simulation of elastic waves propagation in large heterogeneous media still remains a challenging task, especially when high frequency ranges are involved and/or when the geometries of the whole domain or of the interior physical interfaces of heterogeneities are complicated. Therefore, in the past two decades, many research works have been carried on the space discontinuous Galerkin (dG) methods that combine the advantages of finite element methods in the space with the possibilities of developing high-order time integration schemes and massively parallel solvers [1–8].

When space dG methods are applied the first-order elastic wave equations, the Jacobian matrices of the hyperbolic system, if expressed in matrix form, are very complicated due to the fourth-order Hooke elastic tensor even in the isotropic case and are different in 2D and 3D cases [2,3]. As a consequence, the physical meaning of the terms involved in the variational dG formulations, more particularly those concerned by numerical fluxes, are hidden and it becomes difficult to develop and analyze numerical fluxes in more complex cases of anisotropic and piecewise homogeneous media. The aim of the present work is to address these concerns by defining a unified and elastic wave oriented variational framework for the first-order velocity–stress wave equations [2,5,9] in the multidimensional and general – anisotropic and piecewise homogeneous – case. An elastic wave oriented eigenanalysis using a tensorial formalism is presented and allows a simple, compact and intrinsic expression of the Jacobian operator of the first-order hyperbolic system in terms of its eigenvalues and eigenmodes. Hence definition of a unified variational framework is possible whatever the space dimension of propagation media and the characteristics of the elastic tensor. An equivalent eigenanalysis is developed in [5], however, it is formulated within matrix forms and is quite complicated.

Three upwind numerical fluxes are considered in the present work. Firstly, the upwind numerical flux proposed by Käser et al. [2], which uses only material properties from the interior of elements and from one side in the case with discontinuous material properties, is merely modified in order to use material properties from elements on both sides of physical interfaces. This flux is called *"uncoupled" upwind flux* herein. Secondly, to reinforce the coupling between the discontinuous material properties on both sides of a physical interface, a natural choice is to use their averages to define numerical fluxes. Both arithmetic and harmonic averages are considered and lead to two other numerical fluxes, called *"coupled" upwind fluxes* herein. Without discontinuous heterogeneities, all these numerical fluxes are identical and exactly solve the Riemann problem on element interfaces, which is no longer true otherwise. In the general case with discontinuous heterogeneities, the use of all the three simplified fluxes needs to be investigated for verification.

Even though space dG methods can be combined successfully with high order time integration schemes, such as the ADER (arbitrary high order derivatives) approach [2,10], only a fourth-order Runge–Kutta method is used in the present work as the time domain solving is not the object of the present study. The obtained dG solver is explicit with a global mass matrix composed by completely uncoupled elementary mass matrices. Consequently, its parallelization based on MPI is straightforward.

The implemented solvers are firstly applied to both 1D and 2D media with only one physical interface. Accuracy of the three numerical fluxes to describe wave reflection, transmission and conversion phenomena is considered.