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Dedicated to Professor Pravir Dutt on the occasion of his 60th birthday.

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Abstract

In this paper, an h/p spectral element method with least-square formulation for parabolic interface problem will be presented. The regularity result of the parabolic interface problem is proven for non-homogeneous interface data. The differentiability estimates and the main stability estimate theorem, using non-conforming spectral element functions, are proven. Error estimates are derived for h and p versions of the proposed method. Specific numerical examples are given to validate the theory.

Keywords: Least-squares method, nonconforming, spectral element method, Linear parabolic interface problems, Sobolev spaces of different orders in space and time

1. Introduction

In this paper, we consider a linear parabolic interface problem of the form

$$\begin{aligned} \mathcal{L}u = u_t - \nabla \cdot (\mathcal{A}\nabla u) &= F \text{ in } (\Omega_1 \cup \Omega_2) \times I, \\ u &= f \text{ on } \Omega \times \{0\} \qquad \text{(initial condition)} \\ u &= g \text{ on } \Gamma \times I, \quad \text{(exterior boundary condition)} \end{aligned} \tag{1.1}$$

which satisfies the interface conditions

$$[u] = q_0 \quad \text{and} \quad [n \cdot \mathcal{A}\nabla u] = q_1 \text{ on } \Gamma_0 \times I,$$

where $n = (n_1, n_2)^T$ is a unit outward normal vector to the interface Γ_0 and $I = (0, T)$. Here Ω and Ω_1 ($\bar{\Omega}_1 \subset \Omega$) are open bounded domains in $\mathbb{R}^2$ with C^2 boundaries $\partial\Omega = \Gamma$ and $\partial\Omega_1 = \Gamma_0$, respectively (see Fig. 1). Further, $\Omega_2 = \Omega \setminus \bar{\Omega}_1$. The symbol $[v]$ denotes the jump of a quantity v across the interface Γ_0, i.e., $[v](x,t) = v_1(x,t) - v_2(x,t), (x,t) \in \Gamma_0 \times I$. Let

$$\mathcal{A} = \begin{cases} \mathcal{A}^1 \text{ in } \Omega_1 \times I, \\ \mathcal{A}^2 \text{ in } \Omega_2 \times I. \end{cases} . \tag{1.2}$$

Then the jump term $n \cdot \mathcal{A}\nabla u$ is defined as follows:

$$[n \cdot \mathcal{A}\nabla u] = n \cdot (\mathcal{A}^1 \nabla u_1 - \mathcal{A}^2 \nabla u_2) \quad \text{on } \Gamma_0 \times I,$$

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