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Brian N. Granzow, Assad A. Oberai, Mark S. Shephard

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# Adjoint-Based Error Estimation and Mesh Adaptation for Stabilized Finite Deformation Elasticity

Brian N. Granzow[b,*], Assad A. Oberai[a], Mark S. Shephard[b]

[a] *Aerospace and Mechanical Engineering University of Southern California, Los Angeles, CA 90089*
[b] *Scientific Computation Research Center Rensselaer Polytechnic Institute 110 8th Street Troy, NY 12180*

**Abstract**

Adjoint-based error estimation provides the ability to approximate the discretization error for a functional quantity of interest, such as point-wise displacements or stresses. Mesh adaptation provides the ability to control the discretization error to obtain more accurate solutions while still remaining computationally feasible. In this paper, we develop an approach for adjoint-based error estimation and mesh adaptation for nonlinear finite deformation elasticity using a mixed stabilized finite element method. We apply our developed technique to a well known test case, the Cook's membrane problem, to validate and demonstrate its effectiveness. We then demonstrate the utility of adjoint-based error estimation and mesh adaptation for a three-dimensional example motivated by the study of a cell embedded in a matrix.

*Keywords:* adjoint, a posteriori, functional, error estimation, adaptation, nonlinear, elasticity, stabilized, finite element

## 1. Introduction

The purpose of this paper is to develop an approach for functional error estimation and mesh adaptation using adjoint-based techniques for incompressible finite deformation elasticity. An important scenario where incompressible nonlinear elastic materials are utilized is the study of biological soft tissues [23, 29, 10]. Adjoint-based error estimation provides the ability to approximate discretization errors for a functional quantity of interest (QoI) [40, 5, 16, 30, 31, 7, 3], such as point-wise displacements or stresses, or the integrated displacement over a sub-domain. Mesh adaptation utilizes local information obtained from error estimates to control discretization errors by adaptively modifying the computational mesh.

Previously, in the context of solid mechanics, adaptive adjoint-based error estimation has been used to study linear elasticity in two [34, 38, 17] and three [14] dimensions, two [35, 36] and three [15] dimensional elasto-plasticity, two dimensional thermoelasticity [32], two dimensional nonlinear elasticity [22], and two dimensional hyperelasticity [43]. In the vast majority of the previous literature, mesh adaptation is performed with structured adaptive mesh refinement using quadrilateral or hexahedral elements. However, for complex geometries such as those that arise in the study of biological tissues, mesh generation and mesh adaptation are reliable, robust, and scalable for simplical elements. This motivates us to consider triangular and tetrahedral elements.

It is well known that solid mechanics problems with incompressibility constraints perform poorly with linear displacement-based Galerkin finite element methods when using simplical elements. This motivates

*Corresponding author, brian.granzow@gmail.com