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On the Global Interpolation of Motion

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Abstract

Interpolation of motion is required in various fields of engineering such as computer animation and vision, trajectory planning for robotics, optimal control of dynamical systems, or finite element analysis. While interpolation techniques in the Euclidean space are well established, general approaches to interpolation on manifolds remain elusive. Interpolation schemes in the Euclidean space can be recast as minimization problems for weighted distance metrics. This observation allows the straightforward generalization of interpolation in the Euclidean space to interpolation on manifolds, provided that a metric of the manifold is defined. This paper proposes four metrics of the motion manifold: the matrix, quaternion, vector, and geodesic metrics. For each of these metrics, the corresponding interpolation schemes are derived and their advantages and drawbacks are discussed. It is shown that many existing interpolation schemes for rotation and motion can be derived from the minimization framework proposed here. The problems of averaging of rotation and motion can be treated easily within the same framework. Both local and global interpolation problems are addressed. The proposed interpolation framework can be used with any suitable set of basis functions. Examples are presented with Chebyshev spectral, Fourier spectral, and B-spline basis functions. This paper also introduces one additional approach to the interpolation of motion based on the interpolation of its derivatives. While this approach provides high accuracy, the associated computational cost is high and the approach cannot be used in multi-variable interpolation easily.

1 Introduction

Interpolation of rotation and rigid-body motion is a common problem that arises in various fields of engineering such as computer animation and vision, trajectory planning for robotics, or optimal control of dynamical systems. In the area of multibody system dynamics, interpolation of motion is required when implementing finite element and spectral methods. Interpolation can be viewed as a mathematical operation that approximates a continuous field based on its discrete values at a set of points. For instance, in finite element methods, the displacement field is known at the nodes of the element and interpolation is required to evaluate the displacement and strain fields at the Gauss points; this process can be viewed as a "local interpolation" within the element. On the other hand, in spectral methods, the displacement field is known at the grid points and interpolation is required to evaluate strains at the same grid points; this process can be viewed as a "global interpolation" over the entire domain and is expected to yield the exponential convergence property of spectral methods. Of course, the nature of the field to be interpolated depends on the specific application. In this paper, the interpolation of both rotation and rigid-body motion is treated in a unified manner and is referred to as "the interpolation of motion."

Interpolation techniques in Euclidean space are well established. Given a set of vectors, $\underline{x}_k \in \mathbb{R}^m$, located at grid points $\underline{\eta}_k$, $k = 0, 1, \ldots, N$, classical interpolation schemes define the interpolated