Accepted Manuscript

Analysis and mixed-primal finite element discretisations for stress-assisted diffusion problems

Gabriel N. Gatica, Bryan Gomez-Vargas, Ricardo Ruiz-Baier

PII: S0045-7825(18)30167-1
DOI: https://doi.org/10.1016/j.cma.2018.03.043
Reference: CMA 11849

To appear in: *Comput. Methods Appl. Mech. Engrg.*

Received date : 4 December 2017
Revised date : 20 March 2018
Accepted date : 26 March 2018

Please cite this article as: G.N. Gatica, B. Gomez-Vargas, R. Ruiz-Baier, Analysis and mixed-primal finite element discretisations for stress-assisted diffusion problems, *Comput. Methods Appl. Mech. Engrg.* (2018), https://doi.org/10.1016/j.cma.2018.03.043

Analysis and mixed-primal finite element discretisations for stress-assisted diffusion problems

Gabriel N. Gatica[a], Bryan Gomez-Vargas[a,b], Ricardo Ruiz-Baier[c]

[a] *CI²MA and Departamento de Ingeniería Matemática, Universidad de Concepción, Casilla 160-C, Concepción, Chile.*
[b] *Sección de Matemática, Sede Occidente, Universidad de Costa Rica, San Ramón de Alajuela, Costa Rica.*
[c] *Mathematical Institute, University of Oxford, A. Wiles Building, Woodstock Road, Oxford OX2 6GG, United Kingdom.*

Abstract

We analyse the solvability of a static coupled system of PDEs describing the diffusion of a solute into an elastic material, where the process is affected by the stresses exerted in the solid. The problem is formulated in terms of solid stress, rotation tensor, solid displacement, and concentration of the solute. Existence and uniqueness of weak solutions follow from adapting a fixed-point strategy decoupling linear elasticity from a generalised Poisson equation. We then construct mixed-primal and augmented mixed-primal Galerkin schemes based on adequate finite element spaces, for which we rigorously derive a priori error bounds. The convergence of these methods is confirmed through a set of computational tests in 2D and 3D.

Keywords: Linear elasticity, stress-assisted diffusion, mixed-primal formulation, fixed-point theory, finite element methods, a priori error bounds
2000 MSC: 65N30, 65N12, 65N15, 76R05, 76D07

1. Introduction

This work is motivated by the mathematical and numerical investigation of stress-enhanced diffusion processes in deformable solids. Starting from the early works by e.g. Truesdell [33], Podstrigach [27], or Aifantis [2], a number of applicative studies and different models have been developed. Many of these contributions have focused on the modelling of hydrogen diffusion in metals [32], damage of electrodes in lithium ion batteries [5], sorption in fibre-reinforced polymeric materials [30], drying of liquid paint layers [34], gels and general-purpose solute penetration [21, 35], anisotropy of cardiac dynamics [9], and several other effects. Irrespective of the specific interaction under consideration, the assumptions in these models convey that the species diffuses on the elastic medium obeying a Fickean law enriched with additional contributions arising from local effects by exerted stresses.

Although there exist numerous advances on the modelling considerations for stress-assisted and strain-assisted diffusion problems, their counterparts from the viewpoint of mathematical and numerical analysis are still far behind. A few punctual references include the study of plane steady solutions [20], asymptotic analysis [11, 34], and the very recent general well-posedness theory for static and transient problems in a primal formulation, developed in [24]. Our goal at this stage is to focus on a simple stationary problem that represents the main ingredients of diffusion-deformation interaction models where the Cauchy stress acts as a coupling variable. We will concentrate on the regime of linear elasticity, and we will further

Email addresses: ggatica@ci2ma.udec.cl (Gabriel N. Gatica), bryan.gomezvargas@ucr.ac.cr (Bryan Gomez-Vargas), ruizbaier@maths.ox.ac.uk (Ricardo Ruiz-Baier)
Funding: This work was partially supported by CONICYT-Chile through BASAL project CMM, Universidad de Chile, and the Becas-Chile Programme for foreign students; by Centro de Investigación en Ingeniería Matemática (CI²MA), Universidad de Concepción; and by the EPSRC through the research grant EP/R00207X/1.