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Unified polynomial expansion for interval and random response analysis of uncertain structure–acoustic system with arbitrary probability distribution

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Highlights

- Unified polynomial expansion is established for interval/random models.
- Arbitrary polynomial chaos is extended for interval and hybrid uncertain analysis.
- The method is applied to uncertain problem with complex probability distribution.
- The proposed method has been compared with the hybrid perturbation method.
- The proposed method has been compared with several other polynomial chaos methods.

Abstract

For structure–acoustic system with uncertainties, the interval model, the random model and the hybrid uncertain model have been introduced. In the interval model and the random model, the uncertain parameters are described as either the random variable with well defined *probability density function* (PDF) or the interval variable without any probability information, whereas in the hybrid uncertain model both interval variable and random variable exist simultaneously. For response analysis of these three uncertain models of structure–acoustic problem involving arbitrary PDFs, a unified polynomial expansion method named as the *Interval and Random Arbitrary Polynomial Chaos method* (IRAPCM) is proposed. In IRAPCM, the response of the structure–acoustic system is approximated by APC expansion in a unified form. Particularly, only the weight function of polynomial basis is required to be changed to construct the APC expansion for the response of different uncertain models. Through the unified APC expansion, the uncertain properties of the response of three uncertain models can be efficiently obtained. As the APC expansion can provide a free choice of the polynomial basis, the optimal polynomial basis for the random variable with arbitrary PDFs can be obtained by using the proposed IRAPCM. The IRAPCM has been employed to solve a mathematical problem and a structure–acoustic problem, and the effectiveness of the unified IRAPCM for response analysis of three uncertain

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models is demonstrated by fully comparing it with the hybrid first-order perturbation method and several existing polynomial chaos methods.

Keywords: Interval model; Random model; Hybrid uncertain model; Arbitrary polynomial chaos; Gauss integration; Structure–acoustic system

1. Introduction

The response analysis of structural–acoustic system is a key procedure for the control and optimization of the vibration and noise behaviors of engineering products, such as automobiles, steamships, aircrafts, submarines and spacecrafts. Traditional methods for response analysis of structural–acoustic system are deterministic numerical methods by assuming that all input parameters are fixed [1]. However, uncertainties related to material properties, boundary conditions and surrounding environment are unavoidable in the real engineering practices. Without considering these uncertainties, the results obtained by using deterministic numerical methods may be unreliable. Therefore, there is a growing interest for developing numerical methods for the response analysis of structural–acoustic system with uncertainties [2–6].

The most widely used technique for uncertainty quantification is the probabilistic method, in which the uncertain parameter is represented by the random variable with well defined *probability density function* (PDF). During past decades, lots of methods have been proposed for random uncertainty quantification, such as the Monte Carlo method [7–9], the perturbation probabilistic method [10–13] and the polynomial chaos method [14,15]. Among these methods, the Monte Carlo method is the simplest and the most versatile method for uncertain problems. However, the Monte Carlo method suffers from tremendous computational cost for large-scale engineering systems [7]. The perturbation probabilistic method is a very efficient way for random analysis, but it is only accurate for uncertain problems with small uncertainty level [10]. The polynomial chaos method is proposed based on orthogonal polynomial theory, which is free from small perturbation assumption and the efficiency is much higher than Monte Carlo method [14]. Thus, the polynomial chaos method has been widely used to solve random engineering problems [16–18].

The probabilistic method is established based on the condition that the precise probability distribution is obtained. However, at the early stage of design, the PDF of random variables may be not available due to the limited information. To model the uncertain problems with limited information, various of non-probabilistic mathematical frameworks have been developed, such as the interval analysis [19–21], the fuzzy theory [22,23], the evidence theory [24–26] and the p-box set [27,28]. All these non-probabilistic mathematical frameworks have their own merit in application. The fuzzy theory is an effectively technique to model the subjective probability derived from the expert opinions. The evidence theory and the p-box set are suitable to represent imprecise probability. In the interval analysis, only the lower and upper bounds of an uncertain parameter are required. Thus, the interval analysis is most suitable to describe the uncertainties whose probability information is completely missing. As the determination of bounds for an interval may be easier and more straightforward than the identification of an imprecise probability distribution, the interval analysis is also a popular mathematical framework to deal with the uncertainties in engineering problems. Researches on uncertainty quantification of interval model is rather mature and different methods have been proposed, including the interval perturbation method [29–31], the interval Chebyshev method [32], the interval Legendre method [33], the interval factor method [34], the vertex method [35], the rational expansion method [36,37] and et al. More detailed review of interval methods can be found from Ref. [38].

Obviously, the interval methods and the random methods aforementioned are focused on uncertain problem with either random or interval parameters. However, the random and interval parameters may exist simultaneously in some engineering problems. To represent the hybrid uncertainties, Elishakoff and Colombi developed a hybrid uncertain model, in which some uncertain parameters with well defined PDFs are treated as random variables, whereas the others are described as interval variables [39]. The uncertainty quantification of the hybrid uncertain model is more challenging than the interval uncertainty quantification and the random uncertainty quantification, as the approximation for the response related to different types of uncertainty in the hybrid uncertain model should be properly integrated [40]. Up to now, the studies for uncertainty quantification of the hybrid uncertain model are