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NURBS plasticity: non-associated plastic flow

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Highlights

- NURBS plasticity formulation extended to include non-associated plastic flow.
- Evolution of plastic strain is decoupled from the yield surface normal allowing for more diverse material behaviour.
- Any smooth isotropic convex yield envelope can be represented within the framework.
- NURBS surfaces combined with an implicit backward-Euler-type stress integration algorithm.
- Algorithm inherently stable and efficient as the iterative process starts and remains on the yield surface.

Abstract

This paper extends the non-uniform rational basis spline (NURBS) plasticity framework of Coombs et al. (2016) and Coombs and Ghaffari Motlagh (2017) to include non-associated plastic flow. The NURBS plasticity approach allows any smooth isotropic yield envelope to be represented by a NURBS surface whilst the numerical algorithm (and code) remains unchanged. This paper provides the full theoretical and algorithmic basis of the non-associated NURBS plasticity approach and demonstrates the predictive capability of the plasticity framework using both small and large deformation problems. Wherever possible errors associated with the constitutive formulation are specified analytically and if not numerical analyses provide this information. The rate equations within the plasticity framework are integrated using an efficient and stable implicit stress update algorithm which allows for the derivation of the algorithmic consistent tangent which ensures optimum convergence of the global out of balance force residual when used in boundary value simulations.

The important extension provided by this paper is that the evolution of plastic strain is decoupled from the yield surface normal. This allows the framework to model more realistic material behaviour, particularly in the case of frictional plasticity models where an associated flow rule is known to significantly overestimate volumetric dilation leading to spurious results. This paper therefore opens the door for the NURBS plasticity formulation to be used for a far wider class of material behaviour than is currently possible.

Keywords: Elasto-plasticity; Constitutive modelling; Non-associated flow; Non-uniform rational basis spline (NURBS); Stress integration; Finite-element analysis

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1. Introduction

Constitutive models that provide incremental relationships between stress and strain are essential for boundary value analysis of engineering problems. Within this, one of the most common classes of material behaviour is elasto-plasticity where the elastic region of stress space is bounded by a yield surface. On this yield surface the material will undergo elasto-plastic material behaviour and countless yield envelopes have been proposed since the works of Tresca [1] and von Mises [2]. However, the form of the yield function impacts on the stress integration algorithm which is required to convert the rate form of the plasticity equations into an incremental form that can be used in boundary value simulations (using the finite element method for example). This issue was overcome by the non-uniform rational basis spline (NURBS) plasticity framework of Coombs et al. [3] and Coombs and Ghaffari Motlagh [4] which allowed any smooth isotropic yield envelope to be integrated using the same numerical algorithm. However, [3] and [4] were limited to the case of associated plastic flow where the form of the yield surface governs both the yielding of the material and the evolution of plastic strains. This limits the form of material behaviour that can be predicted and, in the case of frictional plasticity models, leads to a significant overestimation of volumetric dilation. This paper overcomes this limitation by decoupling the evolution of plastic strains from the yield surface normal leading to a non-associated plastic flow constitutive framework.

In this paper we do not attempt to review all of the constitutive models available in the literature, instead an interested reader is referred to the work of Yu [5] for a general review of constitutive models. In the specific area of NURBS plasticity, beyond the work of [3,4], the only other paper that the authors are aware of is that of Coelho et al. [6] who construct NURBS response surfaces in biaxial strain and stress space based on curve fitting to experimental data. This is quite different to the approach of [3,4], and that advocated in this paper, where a NURBS yield envelope is constructed and then used within a conventional plasticity formulation. This approach provides a constitutive formulation which is valid for generalised, six-component, stress and strain space.

The NURBS plasticity formulation is combined with an implicit predictor–corrector stress integration algorithm [7] to provide an incremental relationship between stress and strain. Several papers have compared different stress integration algorithms and, as with the vast array of constitutive models, in this paper we do not attempt to review them. In this case the interested reader is refereed to the works of Anandarajah [8] and Safaei et al. [9], amongst others. The reasons for adopting an implicit algorithm in this paper are twofold: (i) they rigorously enforce the consistency conditions at the updated stress state (and in the case of NURBS plasticity, throughout the process) and (ii) allow for the derivation of the algorithmic consistent tangent that ensures asymptotic quadratic convergence of the global residual when used within a boundary value simulation.

The layout of the paper is as follows, Section 2 provides the theoretical framework for hardening non-associated flow NURBS-based plasticity, including the definition of the NURBS surface and the non-associated flow rule, isotropic hardening through the movement of control points, the form of stress integration used and the technique of energy mapped stress that allows us to interpret the stress integration method as a geometric projection. Section 3 provides details on the numerical implementation including the backward Euler (bE) stress integration process and the algorithmic consistent tangent. Numerical examples are presented in Section 4 and, finally, conclusions are drawn in Section 5.

The majority of the paper is presented in tensor form using index notation, the notable exception is the numerics that are presented in matrix–vector form for ease of implementation. Due to the geometric nature of the method presented in this manuscript, the majority of the paper is presented in principal stress and strain space with the following ordering of the principal stresses

$$\sigma_1 \geq \sigma_2 \geq \sigma_3,$$

with tensile stresses taken as positive. Note that although the equations are presented in principal stress space we can do this without loss of generality of the final result as the principal quantities are simply transferred back to generalised quantities at the end of the algorithm. Generalised, 6-component, stress and strain quantities are denoted using $\hat{(\cdot)}$.

2. Non-associated flow NURBS plasticity

This section provides the essential equations required for an isotropically hardening NURBS surface with non-associated plastic flow. There is significant overlap between the theory presented here and that of Coombs et al. [3] (for perfect plasticity with associated flow) and Coombs and Ghaffari Motlagh [4] (for isotropic hardening with