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Finite element heterogeneous multiscale method for Elastic Waves in Heterogeneous Media

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Abstract

A finite element heterogeneous multiscale method (FE-HMM) is proposed for the simulation of time-dependent elastic waves in a rapidly varying heterogeneous elastic medium. It is based on a standard finite element discretization of an effective wave equation at the macro scale, whose a priori unknown effective material coefficients are computed on sampling domains at the micro scale within each macro finite element. Hence the computational effort becomes independent of the highly heterogeneous elastic medium at the smallest scale. Optimal error estimates and convergence rates in the energy and the L^2 norm are derived, which are explicit in the macro and micro discretization errors. Numerical experiments verify the sharpness of the error bounds and illustrate the versatility of the method for non-periodic, layered or stochastic media.

Keywords: multiscale methods, heterogeneous media, wave equation, linear elasticity, numerical homogenization, upscaling

1. Introduction

The efficient numerical simulation of time-dependent elastic wave phenomena is of fundamental importance in a variety of scientific and engineering applications. Finite element methods (FEM) are becoming increasingly popular because they easily accommodate localized small scale geometric features such as cracks, fractures, pinch-outs, or material interfaces. Thus when combined with local time-stepping strategies [20, 27], high-order FE methods are probably the method of choice for elastic wave propagation [36].

Yet when material heterogeneities not only occur at a scale much smaller than the wave length but also throughout the computational domain, classical finite element (or finite difference) methods become inefficient. As standard FE methods require grid resolution down to the finest scales in the medium, they indeed lead to prohibitively large problem sizes, even though the wave length itself might occur at a more moderate macroscopic scale. Not only for the simulation of seismic waves at the planetary scale but also for the prediction of the complex time-dependent response of engineered composite materials or structures, the need for multiscale strategies becomes all too obvious.

Multiscale methods generally fall into either of two classes: with or without explicit scale separation. Multiscale methods that forgo any underlying assumption of scale separation