

Modelling of strain softening materials based on equivalent damage force

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Abstract

The main aim of the work presented in this paper was treatment of damage and deformation localisation observed in the finite element method (FEM) analysis of strain softening materials combined with local constitutive models where damage is represented using continuum damage mechanics (CDM). The CDM/FEM approach typically suffers from a number of shortcomings, including mathematical (change of the type of partial differential equations leading to ill-posed boundary value problem), numerical (pronounced mesh dependency) and physical (infinitely small softening zone with the zero dissipated energy). The approach proposed here is still based on the local constitutive model including damage, but introduces an alternative representation of damage effects in the system of linear momentum balance equations. The damage effects are included through equivalent damage force (EDF), which contributes to the right-hand side of the momentum balance equations. The main advantages of this approach are that the problem remains well posed, as the type of partial differential equations remains unchanged when the material enters softening; numerical stability is preserved without a need for regularisation measures; and significantly reduced mesh dependency. In addition, the EDF approach can be used in combination with existing local CDM damage models and does not violate symmetry of the material stiffness tensor.

The EDF approach is applicable to modelling of strain softening typically observed in damaged quasi brittle materials such as fibre reinforced composites and concrete.

The EDF model was implemented in the in-house developed coupled FEM-SPH code, where an explicit FEM code is coupled with a stable Total-Lagrange form of SPH. Its performance is demonstrated in the analysis of a dynamic one dimensional (1D) stress wave propagation problem, which was analytically solved by Bazant and Belytschko in 1985. For a range of loading rates that correspond to the material softening regime, the numerical results shown nonlocal character with a finite size of the damaged zone, controlled with the damage characteristic length, which can be experimentally determined and is an input parameter independent of the discretisation density.

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1. Introduction

Strain softening is deterioration of material strength with increasing strain, which is a phenomenon typically observed at a continuum level in damaged quasi brittle materials, including fibre reinforced composites and concrete. It is primarily a consequence of brittleness and heterogeneity of the material. It has been experimentally demonstrated that the strain softening in the material is distributed over a finite region whose size depends on material type, see for instance [1] and references therein. Averaging is an approach to modelling the strain softening where micromechanical damage effects are smeared over the softening zone in Continuum Damage Mechanics (CDM). An example of damage model, that is often used in with the averaging, is the model where degradation of material properties is represented as a loss of effective load-carrying area [2–4].

When local CDM constitutive models are used with the finite element method (FEM), the strain softening leads to numerical instability, as the tangent stiffness tensor (slope of the stress–strain curve in 1D) loses positive definiteness and violates the material stability criterion by Hadamard [5]. Consequently, the underlying initial boundary value problem becomes ill-posed and the continuum solution bifurcates, leading to an infinite number of solutions. In addition, these local CDM models lead to deformation localised in a single element and consequently pronounced sensitivity of the results to the spatial discretisation (mesh density) as already demonstrated in [6]. Localised deformation and mesh sensitivity lead to infinite local strain with mesh refinement (in the limit). This result is non-physical with unrealistic energy dissipation due to damage within a zero volume zone. In summary, strain softening leads to mathematical pathology, in terms of change of the type of partial differential equations (PDE), numerical pathology, in terms of mesh sensitivity, and leads to the physically meaningless results.

The strain-softening instabilities have been of large interest to research in recent decades and have been investigated, among many others in [7–15], leading to a development of a number of regularisation methods, including non-local, gradient-enhanced and viscous methods. These methods are based on the introduction of a characteristic length scale into constitutive equations through higher-order spatial derivatives or viscous effects, see for instance the models developed by Dillon [16], Bazant [17,18], Aifantis [19,20], Needleman [21–23], Pijaudier-Cabot [24,7], Sluys [25,26], and de Borst [27–29]. These regularisation methods prevent development of the material instability i.e. prevent change of the type of underlying governing equations, which are elliptic partial differential equations in static problems and hyperbolic in dynamic problems. This in turn leads to a well-posed initial boundary value problem. The material characteristic length scale defines the size of the area affected by strain-softening enabling physically meaningful and mesh-independent finite element solutions.

Despite the evident success of regularisation methods in the field of strain-softening instabilities, research has been almost exclusively focused on these methods and, to date, there has been little research into solutions based on local constitutive equations. However, this might be of interest to users of strain-softening models as regularisation methods necessitate an increased understanding of the underlying strain-softening problem, definition of the characteristic length for the material of interest and make the application of regularisation methods numerically more expensive. More importantly, a suitable description of damage effects in a continuum combined with CDM, allows for more flexibility in formulation of constitutive models and related material experimental characterisation. We, among a number of other researchers, demonstrated that strain-softening results in numerical instability and highly mesh sensitivity in FEM analyses [6]. The aim of the work presented in this paper was development of an alternative new approach to modelling damage in strain softening materials within the FEM framework and based on the local constitutive equations. The model developed is called equivalent damage force (EDF). The key feature of this approach is that the material damage effects are represented as a force on the right-hand side of the balance of linear momentum equation. The proposed EDF method maintains a well-posedness of initial boundary value problems and, therefore, does not require any regularisation measures within constitutive equations in modelling strain-softening materials. In addition, the method can be combined with any CDM local damage law, providing the mesh independent stable solutions.

This paper consists of five sections. Following the introduction of the strain softening problem and associated issues, a benchmark dynamic strain softening problem is described in Section 2 with the analytical local and nonlocal solutions proposed by Bazant and Belytschko [30]. The Equivalent Damage Force approach is presented in Section 3, including the derivation of principle equations and model implementation into the in-house coupled FEM–SPH code. The proposed approach is validated against the known analytical solutions in Section 4, with the outcomes of this work summarised in Section 5.

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