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Stabilized Mixed Finite Element Method for the M_1 radiation model

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Abstract

In this work, we present a computational approach for the numerical simulation of thermal radiation. Radiation is modeled by solving a set of two coupled partial differential equations, the so-called M_1 model. A Variational Multiscale method is developed for this system, and tested on some illustrative benchmarks. The question of dealing with heterogeneous physical properties is also considered, and treated by means of an immersed method. It combines a level-set approach for representation of interfaces, mixing laws to build effective physical properties and an anisotropic mesh adaptation process, all these ingredients leading to an accurate description of the interface.

Keywords: Variational Multiscale Method, Stabilized Finite Elements, radiation, Immersed Volume method, Monolithic approach

1. INTRODUCTION

In many high temperature applications, such as industrial furnaces [1, 2, 3], glass treatment [4, 5] nuclear engineering [6], or combustion and flame modeling [7, 8, 9], thermal radiation is the dominant mode of heat transfer. However, the full Radiative Transfer Equation (RTE), due to its seven variables dependency (three of space, two of directions, time and frequency) is, in many situation, too expensive to solve directly (particularly due to the angular dependency). So, in recent years, with the growing power of computers, a great effort has been devoted to derive physically relevant models that are affordable from the numerical point of view (see [10], for instance).

The existing models differ in the way that angular dependency is treated. One of these models is for example the Spherical Harmonic approach, where the angular dependency of the specific intensity is expressed using Legendre polynomials [11]. Another class of methods is that based on direction-discretization approaches, in which the RTE is solved for a discrete set of directions, namely the discrete ordinates [12]; this leads to a system of ordinary partial differential equations, one for each discrete direction. However, in both cases, the price to pay to get a good numerical approximation is a high number of discrete ordinates or a high order Legendre polynomial, both of which come with a very high computational cost. There exist also other approaches, based on a totally different philosophy, like the "Surface-to-Surface" ($S2S$) or the zonal method [13]. The idea here is to compute net radiative exchanges between different parts of the computational domain (surface-surface, surface-volume and volume-volume exchanges). Then, those radiative exchanges are included in the global thermal balance. Within this framework, the radiation part is computed apart from the other physics in coupled problems (flow, heat transfer, turbulence). However, those approaches can become very computationally demanding for unsteady simulations, limiting the range of application of such methods.

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