Available online at www.sciencedirect.com
ScienceDirect
Comput. Methods Appl. Mech. Engrg. 335 (2018) 91–127
Computer methods in applied mechanics and engineering
www.elsevier.com/locate/cma

An *a posteriori*, efficient, high-spectral resolution hybrid finite-difference method for compressible flows

Javier Fernández-Fidalgo*, Xesús Nogueira*, Luis Ramírez, Ignasi Colominas

Universidade da Coruña, Group of Numerical Methods in Engineering, Campus de Elviña, 15071, A Coruña, Spain

Received 28 September 2017; received in revised form 7 February 2018; accepted 9 February 2018
Available online 1 March 2018

Abstract

A high-order hybrid method consisting of a high-accurate explicit finite-difference scheme and a Weighted Essentially Non-Oscillatory (WENO) scheme is proposed in this article. Following this premise, two variants are outlined: a hybrid made up of a Finite Difference scheme and a compact WENO scheme (CRWENO 5), and a hybrid made up of a Finite Difference scheme and a non-compact WENO scheme (WENO 5). The main difference with respect to similar schemes is its *a posteriori* nature, based on the Multidimensional Optimal Order Detection (MOOD) method. To deal with complex geometries, a multi-block approach using Moving Least Squares (MLS) procedure for communication between meshes is used. The hybrid schemes are validated with several 1D and 2D test cases to illustrate their accuracy and shock-capturing properties.

Keywords: High-order schemes; Compressible flows; Overset grids; Finite differences

1. Introduction

The solution of partial differential equations in presence of strong shocks has always been a difficult task. Several techniques have been explored in order to achieve high-order results, with the consequential increase of computational cost. In this article we explore a new hybrid technique based on the *a posteriori* detection paradigm that combines a Finite Difference (FD) scheme and a Weighted Essentially Non-Oscillatory (WENO) scheme. Obviously the methodology presented in this article is not only restricted to the schemes we present, but can also be applied to any combination of finite difference and WENO schemes regardless of their order of accuracy and/or compactness.

The key idea is to combine a fast and accurate scheme that cannot deal with shocks, with a scheme that can deal with them accurately at the expense of a higher computational cost. The scheme that can deal with shocks will only be used on those areas where the fast and accurate scheme is unable to obtain a quality solution. Thus, it is crucial the accurate and reliable detection of the problematic zones.

* Corresponding authors.
E-mail addresses: javier.fernandez1@udc.es (J. Fernández-Fidalgo), xnogueira@udc.es (X. Nogueira).

https://doi.org/10.1016/j.cma.2018.02.013

To that matter, several methods combining FD with WENO schemes have been proposed in the literature. For instance, Costa and Don presented in [1] a hybrid method composed by a sixth order FD scheme and a fifth order WENO scheme for the smooth and discontinuous parts of the solution, respectively. The type of criterion used to switch from one scheme to the other relies on a sensor that detects the discontinuities in advance (*a priori* detection).

The approach proposed by Pirozzoli [2] combines a fifth-order compact upwind algorithm for the smooth parts of the flow with a fifth-order WENO scheme to capture discontinuities. The detection criterion is *a priori* as well, employing a threshold value to distinguish the smooth from the non-smooth zones.

As explained in [3], the drawback of these approaches is that the shock locations are predicted, and related to the *a priori* guesses there can be a loss of efficiency due to over-detection of problematic zones. This happens around critical points, where the denominator of the derivatives approaches zero, and the method switches to the WENO scheme. To decrease the over-dissipation associated with such a degeneration, dimensional parameters, which require tuning for different problems, have been introduced in these *a priori* approaches.

In this work, an *a posteriori* detection criterion based on the Multidimensional Optimal Order Detection (MOOD) method is proposed. The reader is referred to [4,5] for details on the MOOD paradigm.

Our approach differs from the original MOOD because of the underlying schemes we use for the calculations. In the original MOOD method [4], a single scheme ranging from arbitrary high-order to first order is used, so that the reconstruction order of the problematic cells is gradually downgraded up to first order in case all the others attempts have failed. In our approach, we combine two methods of similar order, but with a significant difference in terms of computational cost. The fastest method is not able to deal with shocks, whereas the other method can handle shock waves at the price of increasing the overall computational cost. This conceptual difference is why we do not label the proposed methodology as MOOD, since it is an *a posteriori* approach but the order of the chosen schemes is not downgraded.

In this work we also present an approach based on Moving Least Squares (MLS) to apply the numerical scheme to block-structured meshes.

The structure of the paper is as follows. First, the governing equations and the different numerical methods used are presented. Then, the *a posteriori* detection paradigm is introduced. In Section 6 we present the MLS-based technique for multi-block grid, and then we present some numerical examples to show the accuracy, efficiency and robustness of the proposed hybrid schemes. Finally, conclusions are drawn.

2. Governing equations

The goal of this article is to solve the two-dimensional Euler equations in general coordinates for an inviscid, compressible, Newtonian fluid. Following the general curvilinear transformation $(x, y) \rightarrow (\xi, \eta)$ as in [6], these equations are written in the following strong conservation form:

$$\frac{\partial \hat{\mathbf{U}}}{\partial t} + \frac{\partial \hat{\mathbf{F}}}{\partial \xi} + \frac{\partial \hat{\mathbf{G}}}{\partial \eta} = \mathbf{0} \tag{1}$$

where $\hat{\mathbf{U}}$ denotes the transformed vector of conservative variables, being the original vector $\mathbf{U} = (\rho, \rho u, \rho v, \rho E)^T$ and $\hat{\mathbf{F}}$ and $\hat{\mathbf{G}}$ are the generalized inviscid flux-vectors. Using the same notation as in [7], these vectors can be expressed as

$$\hat{\mathbf{U}} = \frac{1}{J}\begin{pmatrix} \rho \\ \rho u \\ \rho v \\ \rho E \end{pmatrix} \quad \hat{\mathbf{F}} = \frac{1}{J}\begin{pmatrix} \rho \hat{U} \\ \rho u \hat{U} + \xi_x p \\ \rho v \hat{U} + \xi_y p \\ (\rho E + p)\,\hat{U} \end{pmatrix} \quad \hat{\mathbf{G}} = \frac{1}{J}\begin{pmatrix} \rho \hat{V} \\ \rho u \hat{V} + \eta_x p \\ \rho v \hat{V} + \eta_y p \\ (\rho E + p)\,\hat{V} \end{pmatrix} \tag{2}$$

where ρ is the density, u and v are the velocity components along the x and y axes, p is the pressure, and E is the total energy per unit mass expressed as

$$E = \frac{p}{\rho\,(\gamma - 1)} + \frac{1}{2}\left(u^2 + v^2\right) \tag{3}$$

In (3), γ is the ratio of specific heat coefficients of the gas/fluid (for an ideal, monatomic gas, $\gamma = 7/5$). The quantities ξ_x, ξ_y, η_x and η_y are the spatial metrics of the transformation between the physical domain (x, y) and the computational space (ξ, η), where the subscript denotes partial derivation.