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A sparse-grid isogeometric solver

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Abstract

Isogeometric Analysis (IGA) typically adopts tensor-product splines and NURBS as a basis for the approximation of the solution of PDEs. In this work, we investigate to which extent IGA solvers can benefit from the so-called sparse-grids construction in its combination technique form, which was first introduced in the early 90's in the context of the approximation of high-dimensional PDEs.

The tests that we report show that, in accordance to the literature, a sparse-grid construction can indeed be useful if the solution of the PDE at hand is sufficiently smooth. Sparse grids can also be useful in the case of non-smooth solutions when some a-priori knowledge on the location of the singularities of the solution can be exploited to devise suitable non-equispaced meshes. Finally, we remark that sparse grids can be seen as a simple way to parallelize pre-existing serial IGA solvers in a straightforward fashion, which can be beneficial in many practical situations.

Highlights

- A sparse grid version of classical isogeometric solvers is proposed
- The proposed methodology is competitive with standard isogeometric solvers if the solution of the PDE is sufficiently smooth
- The proposed methodology can reuse pre-existing isogeometric solvers almost out-the-box and can be thought as a simple way to parallelize a serial solver
- Radical meshes in the parametric domain can be adopted as a remedy to improve sparse grid convergence for solutions without sufficient regularity on domains with corners.

Keywords: Isogeometric analysis, B-splines, NURBS, sparse grids, combination technique

1. Introduction

Isogeometric analysis (IGA), which was introduced by Hughes et al. [1, 2] in 2005, consists in solving numerically a PDE by approximating its solution with B-splines, Non-Uniform Rational B-Splines (NURBS), and extensions, i.e., with the same basis employed to parametrize the computational geometry with CAD softwares. The method has attracted considerable attention in the engineering computing community, not only because it could simplify the meshing process but also because of other interesting features, including a larger flexibility in the choice of the polynomial degree and regularity of the basis used to approximate the solution, and a more effective error vs. degrees-of-freedom ratio with respect to standard finite element methods; see [3] and references therein.

In this paper, we focus on computational cost efficiency, and in particular on the fact that d-dimensional splines are generated by tensorization of univariate splines, which means that the computational work typically increases exponentially with the dimension d of the problem. Although this phenomenon is somehow

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