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# Refined Isogeometric Analysis for a Preconditioned Conjugate Gradient Solver

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## Abstract

Starting from a highly continuous Isogeometric Analysis (IGA) discretization, refined Isogeometric Analysis (rIGA) introduces $C^0$ hyperplanes that act as separators for the direct LU factorization solver. As a result, the total computational cost required to solve the corresponding system of equations using a direct LU factorization solver dramatically reduces (up to a factor of 55) [1]. At the same time, rIGA enriches the IGA spaces, thus improving the best approximation error. In this work, we extend the complexity analysis of rIGA to the case of iterative solvers. We build an iterative solver as follows: we first construct the Schur complements using a direct solver over small subdomains (macro-elements). We then assemble those Schur complements into a global skeleton system. Subsequently, we solve this system iteratively using Conjugate Gradients (CG) with an incomplete LU (ILU) preconditioner. For a 2D Poisson model problem with a structured mesh and a uniform polynomial degree of approximation, rIGA achieves moderate savings with respect to IGA in terms of the number of Floating Point Operations (FLOPs) and computational time (in seconds) required to solve the resulting system of linear equations. For instance, for a mesh with four million elements and polynomial degree $p = 3$, the iterative solver is approximately 2.6 times faster (in time) when applied to the rIGA system than to the IGA one. These savings occur because the skeleton rIGA system contains fewer non-zero entries than the IGA one. The opposite situation occurs for 3D problems, and as a result, 3D rIGA discretizations provide no gains with respect to their IGA counterparts when considering iterative solvers.

*Keywords:* Isogeometric Analysis (IGA), Finite Element Analysis (FEA), refined Isogeometric Analysis (rIGA), solver-based discretization, iterative solvers, Conjugate gradient, Incomplete LU factorization, k-refinement.

## 1. Introduction

Galerkin-based discretizations such as Finite Element Analysis (FEA) and Isogeometric Analysis (IGA) are commonly employed nowadays to solve numerical problems governed by partial differential equations (PDEs) [2–16]. These methods employ a variational formulation with trial and test functions defined as a linear combination of basis functions to build a discretization of the governing PDEs.

Isogeometric Analysis (IGA) defines the geometry using conventional Computed-Aided Design (CAD) functions and, in particular, non-uniform rational basis spline (NURBS) [2]. These functions represent complex geometries commonly found in engineering design and are capable of preserving exactly the geometry description under refinement as required in the analysis. Moreover, the use of NURBS as basis functions is compatible with the isoparametric

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