Accepted Manuscript

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PII: S0045-7825(18)30036-7
DOI: https://doi.org/10.1016/j.cma.2018.01.034
Reference: CMA 11754

To appear in: *Comput. Methods Appl. Mech. Engrg.*

Received date : 30 August 2017
Revised date : 16 January 2018
Accepted date : 18 January 2018

Please cite this article as: A. Haber, M. Verhaegen, Sparsity preserving optimal control of discretized PDE systems, *Comput. Methods Appl. Mech. Engrg.* (2018), https://doi.org/10.1016/j.cma.2018.01.034

Sparsity Preserving Optimal Control of Discretized PDE Systems

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Abstract

We focus on the problem of optimal control of large-scale systems whose models are obtained by discretization of partial differential equations using the Finite Element (FE) or Finite Difference (FD) methods. The motivation for studying this pressing problem originates from the fact that the classical numerical tools used to solve low-dimensional optimal control problems are computationally infeasible for large-scale systems. Furthermore, although the matrices of large-scale FE or FD models are usually sparse banded or highly structured, the optimal control solution computed using the classical methods is dense and unstructured. Consequently, it is not suitable for efficient centralized and distributed real-time implementations. We show that the *a priori* sparsity patterns of the exact solutions of the generalized Lyapunov equations for FE and FD models are banded matrices. The *a priori* sparsity pattern predicts the structure (non-zero entries) of the exact solution. We furthermore show that for well-conditioned problems, the *a priori* sparsity patterns are not only banded but also sparse matrices. On the basis of these results, we develop two computationally efficient methods for computing sparse approximate solutions of generalized Lyapunov equations. Using these two methods and the inexact Newton method, we show that the solution of the generalized Riccati equation can be approximated by a banded matrix. This enables us to develop a novel computationally efficient optimal control approach that is able to preserve the sparsity of the control law. We perform extensive numerical experiments that demonstrate the effectiveness of our approach.

Keywords: Finite Element Methods; Optimal control; Large-scale systems; Riccati equation; Lyapunov equation.

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