Accepted Manuscript

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PII: S0045-7825(18)30060-4
DOI: https://doi.org/10.1016/j.cma.2018.01.052
Reference: CMA 11772

To appear in: *Comput. Methods Appl. Mech. Engrg.*

Received date : 4 October 2017
Revised date : 29 January 2018
Accepted date : 30 January 2018

Improved Conditioning of Isogeometric Analysis Matrices for Trimmed Geometries

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Abstract

A stable basis for isogeometric analysis of trimmed models is obtained by combining extended B-splines with truncated hierarchical B-splines. While extended B-splines guarantee that the condition number of system matrices is independent of the location of a trimming curve, local refinement is used to improve the robustness of the procedure and the accuracy of the numerical results. The present extended B-spline construction works in the context of Galerkin and collocation methods. The paper focuses on the latter and introduces a new collocation scheme for truncated hierarchical B-splines. A proper transition between refinement levels is assured by a novel balancing algorithm that employs a simple criterion. The enhanced performance of the locally refined stabilization is verified by scalar Laplace and linear elasticity problems analyzed by a collocation based isogeometric boundary element method. The proposed approach yields excellent results and requires few refinement levels to improve the stabilization procedure and accuracy along trimming curves.

Keywords: Trimming, Extended B-splines, WEB-splines, THB-splines, BEM, Collocation

1. Introduction

Trimming is an ubiquitous procedure in Computer Aided Design (CAD) systems. It is used to represent models of arbitrary topological genus simply by displaying specified areas $\mathcal{A}^{\mathrm{v}}$ of a tensor product surface. In essence this is an optical illusion since the underlying mathematical description remains unaltered. Consider the surface depicted in Figure 1: instead of defining a set of control points and a corresponding parameterization that is aligned with the boundaries of the object sought 1(c), a tensor product surface 1(a) is used as a latent representation in the background and a trimming curve, defined in its parameter space 1(b), indicates the final shape shown by the graphics system. Unfortunately, trimming procedures lead to several severe problems within CAD and downstream applications [33]. The present paper is concerned with one of these issues – the stability of a trimmed basis. Trimming procedures may compromise the conditioning of system matrices if only a small portion of a associated basis function's support is within $\mathcal{A}^{\mathrm{v}}$. This problem arises as soon as a trimmed basis is incorporated into an analysis, where trimming is no longer a primarily visual process.

An elegant way to re-establish the stability of a trimmed basis is the utilization of *extended B-splines*. Originally, these splines have been developed in the context of a B-spline based fictitious domain method referred to as WEB-splines [21, 22, 23, 24]. Most extended B-spline applications are tailored to Galerkin formulations based on uniform parameter spaces, but the concept is not restricted to this setting. Non-uniform extended B-splines have been introduced in [22]. Recently, this stabilization has also been applied to collocation schemes for uniform [2] and non-uniform [35] parameter spaces. Regarding the extended

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