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The use of the local truncation error to improve arbitrary-order finite elements for the linear wave and heat equations

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The use of the local truncation error to improve arbitrary-order finite elements for the linear wave and heat equations.

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Abstract

The local truncation error in space and time can be efficiently used for the analysis and the increase in accuracy of the linear and high-order finite elements in the 1-D, 2-D and 3-D cases on uniform and non-uniform meshes. Several applications of the local truncation error are considered in the paper. It is proven that for the 1-D wave equation with a piece-wise constant wave velocity, the local truncation error is zero if the linear finite elements with the element size proportional to the wave velocity, the lumped mass matrix and the central-difference method with the time increments equal to the stability limit are used. It is shown in the 1-D and multidimensional cases that the optimal lumped mass matrix can be calculated by the minimization of the order of the local truncation error and yields the maximum possible order of accuracy. The minimization of the order of the local truncation error allows us to develop the linear finite elements and the isogeometric high-order elements with improved accuracy; i.e., accuracy is improved from order $2p$ (the conventional elements) to order $4p$ (the new elements) where p is the order of the polynomial approximations. New high-order boundary conditions are developed in order to keep a high-order accuracy of the developed technique. The new elements can be equally applied to linear wave propagation and heat transfer problems. It is also shown that non-uniform meshes may lead to inaccurate results due to the increase in the local truncation error. The difference in accuracy between the quadrilateral and triangular linear elements is analyzed with the suggested approach. The presented numerical examples are in good agreement with the theoretical results. The approach considered in the paper can be easily applied to the analysis of different aspects of finite elements techniques as well as other numerical approaches.

Keywords: local truncation error, improved accuracy, finite and isogeometric elements, high-order boundary conditions, wave propagation, heat transfer, lumped mass matrix

This study was first motivated by the analysis and the improvement of accuracy of the linear and high-order finite element techniques applied to wave propagation. The numerical dispersion error is considered as one of the most popular and efficient tools for the analysis of such problems; see [1-30] and many others. The application of the numerical dispersion error is based on the existence of harmonic solutions for an infinite domain for a system of partial differential equations as well as for a discretized system of these equations. However, in many cases, the corresponding harmonic solutions do not exist; e.g., in the case of non-zero load vector of the system of partial differential equations, complicated material properties, non-uniform meshes and many other cases. Therefore, finding more general tools for the accuracy analysis of numerical techniques is a very important task.

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