Overcoming volumetric locking in material point methods

William M. Coombs*, Tim J. Charlton, Michael Cortis, Charles E. Augarde

Department of Engineering, Durham University, Science Site, South Road, Durham, DH1 3LE, UK

Highlights

- The proposed formulation overcomes finite deformation volumetric locking for near-incompressible non-linear solid mechanics.
- The method is applicable to all existing material point methods.
- No restriction is placed on the constitutive model.
- Quasi-static implicit implementation ensures asymptotic quadratic convergence.
- The formulation reduces spurious stress oscillations.

Abstract

Material point methods suffer from volumetric locking when modelling near incompressible materials due to the combination of a low-order computational mesh and large numbers of material points per element. However, large numbers of material points per element are required to reduce integration errors due to non-optimum placement of integration points. This restricts the ability of current material point methods in modelling realistic material behaviour.

This paper presents for the first time a method to overcome finite deformation volumetric locking in standard and generalised interpolation material point methods for near-incompressible non-linear solid mechanics. The method does not place any restriction on the form of constitutive model used and is straightforward to implement into existing implicit material point method codes. The performance of the method is demonstrated on a number of two and three-dimensional examples and its correct implementation confirmed through convergence studies towards analytical solutions and by obtaining the correct order of convergence within the global Newton–Raphson equilibrium iterations. In particular, the proposed formulation has been shown to remove the over-stiff volumetric behaviour of conventional material point methods and reduce stress oscillations. It is straightforward to extend this approach to other material point methods and the presented formulation can be incorporated into all existing material point methods available in the literature.

---

* Corresponding author.
  E-mail address: w.m.coombs@durham.ac.uk (W.M. Coombs).

1. Introduction

Lagrangian mesh-based methods dominate engineering numerical computations in solid mechanics. However, for problems involving large deformations there are issues with pure Lagrangian formulations related to mesh distortion which impact on the accuracy and stability of the methods. With these methods it is therefore necessary to re-mesh and map state variables between the discretisations. One alternative numerical technique that has been demonstrated to be applicable to problems involving very large deformations is the material point method. The material point method was developed by Sulsky et al. [1] as a solid mechanics extension to the fluid implicit particle method [2] which itself was developed from the particle-in-cell method [3]. In the material point method, computations take place on a background grid but the calculations are based on information (stress, state variables, etc.) held at material points that are convected through the background grid as the material deforms. This allows computations to take place on an undistorted background mesh whilst modelling problems involving very large deformations. The simplest way to summarise the material point method is: a finite element method where the integration points (material points) are allowed to move independently from the mesh.

Allowing the material points to move through the background grid reduces the accuracy of the integration required to map information between the material points and the background grid [4]. Therefore, material point methods typically use more material points per element than would be adopted if the elements were integrated using Gauss–Legendre quadrature. Combining this with the fact that material point methods generally use a low order background grid (bi-linear quadrilaterals or tri-linear hexahedrals are a common choice) means that the method is susceptible to volumetric locking (resulting in over-stiff behaviour) when modelling near-incompressible materials. This volumetric locking is caused by excessive constraints placed on an element's deformation by the points used to integrate the stiffness of the element. That is, the constitutive model will require near-isochoric behaviour at the integration (or material) point's location within the element and each of these points places a constraint on the deformation of the element. At a specific number of points the element will lock, resulting in over-stiff behaviour, where the number of points to cause locking is linked to the basis of the element.

A common (and successful) technique to avoid volumetric locking in finite element methods is to use higher order elements with reduced Gaussian integration. However, this is not viable in the material point method, primarily due to the fact that at each step of a material point method analyses it is not known how many material points will be in any given element. In the context of finite deformation solid mechanics, a number of formulations have been proposed to overcome volumetric locking within finite elements, these include: mixed variational methods [5], mixed displacement-pressure formulations [6], geometrically non-linear $\bar{B}$ approaches [7], enhanced assumed strain elements [8–10], co-rotational approaches [11], $\bar{F}$ formulations [12,13] and finite deformation selective reduced integration methods [14], amongst others. See de Souza Neto et al. [15] or de Borst et al. [16] for more detailed reviews of the available methods.

To date the issue of volumetric locking in material point methods has mainly been focused on the analysis of fluid mechanics problems (see for example Zhang et al. [17] and the references contained within), with the notable exception of Mast et al. [18]. Mast et al. [18] investigated the issue of kinematic (volumetric and shear) locking in the material point method and developed a Hu–Washizu multi-field variational principle based approach which introduces independent approximations for the volumetric and the deviatoric components of the strain and stress fields. Although the approach has been shown to overcome volumetric locking in dynamic fluid and solid mechanics problems, it significantly increases the size of the linear system, introduces additional non-physical variables and was only demonstrated for linear material behaviour.

This paper presents for the first time a method to overcome finite deformation volumetric locking in standard and generalised interpolation material point methods for near-incompressible non-linear solid mechanics. In the standard material point method the material points act as discrete lumped masses and only interact with the element that they are located within whereas in the generalised interpolation method each material point has an associated domain which interacts with any elements that it overlaps. To overcome the issue of volumetric locking we adopt the $\bar{F}$ approach of de Souza Neto et al. [12] for the following reasons: (i) unlike mixed approaches it does not introduce any additional unknowns into the linear system, (ii) it is simple to implement within existing finite element codes (and therefore material point codes), (iii) the approach can be used with any constitutive model, (iv) it does not introduce any additional *tuning* parameters into the code and (v) it does not introduce any additional material points to track the volumetric behaviour.