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Recovered Finite Element Methods

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Abstract

We introduce a family of Galerkin finite element methods which are constructed via recovery operators over element-wise discontinuous approximation spaces. This new family, termed collectively as *recovered finite element methods (R-FEM)* has a number of attractive features over both classical finite element and discontinuous Galerkin approaches, most important of which is its potential to produce stable conforming approximations in a variety of settings. Moreover, for special choices of recovery operators, R-FEM produces the same approximate solution as the classical conforming finite element method, while, trivially, one can recast (primal formulation) discontinuous Galerkin methods. A priori error bounds are shown for linear second order boundary value problems, verifying the optimality of the proposed method. Residual-type a posteriori bounds are also derived, highlighting the potential of R-FEM in the context of adaptive computations. Numerical experiments highlight the good approximation properties of the method in practice. A discussion on the potential use of R-FEM in various settings is also included.

1 Introduction

Galerkin procedures are extremely popular in numerical approximation of solutions to initial and/or boundary value problems for partial differential equations (PDEs). The most used families of Galerkin procedures are the (classical, conforming or non-conforming) finite element (FEM) and, more recently, discontinuous Galerkin (dG) finite element families of methods. Roughly speaking, FEM are attractive for their simplicity and robustness, especially in structural mechanics and heat flow simulations, owing to their variational interpretation and origins; dG methods, on the other hand, are popular in fluid flow and fast convection/transport simulations, due to their superior numerical stability properties, stemming from the ability to incorporate general numerical flux functions seamlessly.

FEM typically incorporate (approximate) continuity/conformity of the state variable(s) and/or of some moments directly into the finite element space in order to imitate the respective properties of the underlying continuous problem. As a result, FEM's approximation capabilities have to be assessed for each choice of finite element spaces. At the other end of the spectrum, dG methods typically employ element-wise discontinuous approximation spaces whose approximation properties are clear; the continuity/conformity of the state variable(s) and/or of some moments is enforced weakly via numerical flux functions incorporated in the variational formulation of the dG method. Consequently, the approximation capabilities of dG schemes is only

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