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Isogeometric Boundary Element Analysis of steady incompressible viscous flow, Part 2: 3-D problems

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Abstract

This is a sequel to a previous paper (Beer et al., 2017) [1] where a novel approach was presented to the 2-D Boundary Element analysis of steady incompressible viscous flow. Here the method is extended to three dimensions. NURBS basis functions are used for describing the geometry of the problem and for approximating the unknowns. In addition, the arising volume integrals are treated differently to published work and volumes are described by bounding NURBS surfaces instead of cells and only one mapping is used. The advantage of the present approach is that complex boundary shapes can be described with very few parameters and that no generation of cells is required. For the solution of the non-linear equations full and modified Newton–Raphson methods are used. A comparison of the two methods is made on the classical example of a forced cavity flow, where accurate two-dimensional solutions are available in the literature. Finally, it is shown on a practical example of an airfoil how more complex boundary shapes can be approximated with few parameters and a solution obtained with a small number of unknowns.

Keywords: BEM; Isogeometric analysis; Flow; Incompressible

1. Introduction

Numerous approaches to numerically solve incompressible viscous flow problems can be found in the literature. Most publications use domain methods such as Finite Difference, Finite Elements or Finite Volumes (see for example [2]). A classical example to test published numerical methods is the forced flow in a cavity and very accurate solutions are available for comparison. In [3] for example an extremely fine finite difference mesh is used for the solution. We will use these solutions to compare our results.

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Here we use the Boundary Element method (BEM). The advantage of this method is that, for linear problems, unknowns only exist on the boundary and that the solution inside the domain satisfies the governing differential equations exactly. For nonlinear problems, such as the one discussed here, volume integrals arise which need to be dealt with and this will be discussed in more detail in the successive sections (see also [1]).

1.1. The boundary element method for viscous flow

The governing differential equations for steady incompressible viscous flow can be developed from the laws governing the conservation of mass and momentum and assume the following differential forms:

$$\begin{aligned} \frac{\partial u_j}{\partial x_j} &= 0 \\ \mu \frac{\partial^2 u_i}{\partial x_j \partial x_j} - \frac{\partial p}{\partial x_i} - \rho u_j \frac{\partial u_i}{\partial x_j} &= 0 \end{aligned} \tag{1}$$

where x_i is an Eulerian coordinate, u_i is a velocity vector, p is the pressure, ρ the mass density and μ the viscosity.

The requirement for the BEM is the existence of fundamental solutions of the differential equations. These solutions can be found for the nonlinear equations (1) only if we consider the non-linear terms as body forces. We rewrite the equations as:

$$\begin{aligned} \frac{\partial u_j}{\partial x_j} &= 0 \\ \mu \frac{\partial^2 u_i}{\partial x_j \partial x_j} - \frac{\partial p}{\partial x_i} + f_i &= 0 \end{aligned} \tag{2}$$

with

$$f_i = -\rho u_j \frac{\partial u_i}{\partial x_j}. \tag{3}$$

Fundamental solutions of Eqs. (2) can now be obtained for an infinite domain by substituting the Dirac-Delta function for the body force.

We define fluid stresses as:

$$\sigma_{ij} = \mu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \tag{4}$$

and the resulting tractions on boundary S:

$$t_i = \sigma_{ij} n_j - p n_i \tag{5}$$

where n_i is the unit vector normal to the boundary. Using the reciprocal theorem, the following integral equation is obtained (see for example [4] and [5]):

$$\begin{aligned} c_{ij}(y)\, \dot{u}_j(y) = &\int_S \left[U_{ij}(y,x)\, t_j(x) - T_{ij}(y,x)\, \dot{u}_j(x) \right] dS(x) \\ &+ \int_{V_0} U_{ij}(y,\bar{x})\, f_j(\bar{x}) dV_0(\bar{x}) \end{aligned} \tag{6}$$

where $c_{ij}(y)$ is an integral free term, depending on the shape of the boundary, V_0 is the subdomain where body forces are present and S_0 is its boundary. $\bar{x}$ specifies a point inside V_0 and $\dot{u}_i$ is the velocity perturbation, i.e. the total velocity can be written as:

$$u_i(x) = \dot{u}_i(x) + u_i^0(x) \tag{7}$$

with u_i^0 the free stream velocity. The values associated with body forces are given by:

$$\begin{aligned} b_{ik}^0 &= \rho\, u_k(\bar{x})\, \dot{u}_i(\bar{x}) \\ t_i^0 &= b_{ik}^0\, n_k \end{aligned} \tag{8}$$

$U_{ij}(y,x)$ and $T_{ij}(y,x)$ are fundamental solutions for the velocity and traction at point x due to a source at point y.