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Extreme value oriented random field discretization based on an hybrid polynomial chaos expansion — Kriging approach

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Highlights

- We seek random fields metamodels which are accurate around their extreme values.
- The metamodel can accurately represent the global extremum for any realization.
- To achieve this an adaptive discretization and enrichment strategy is proposed.
- Original random field representation combing polynomial chaos expansion and Kriging.
- Additional dimensional reduction achieved by Karhunen Loeve decomposition.
- Three example problems, including the aerodynamic design of an aircraft wing.

Abstract

This article addresses the characterization of extreme value statistics of continuous second order random field. More precisely, it focuses on the parametric study of engineering models under uncertainty. Hence, the quantity of interest of this model is defined on both a parametric space and a stochastic space. Moreover, we consider that the model is computationally expensive to evaluate. For this reason it is assumed that uncertainty propagation, at a single point of the parametric space, is achieved by polynomial chaos expansion. The main contribution of the present study is the development of an adaptive approach for the discretization of the random field modeling the quantity of interest. Objective of this new approach is to focus the computational budget over the areas of the parametric space where the minimum or the maximum of the field is likely to be for any realization of the stochastic parameters. To this purpose two original random field representations, based on polynomial chaos expansion and Kriging interpolation, are introduced. Moreover, an original adaptive enrichment scheme based on Kriging is proposed. Advantages of this approach with respect to accuracy and computational cost are demonstrated on several numerical examples. The proposed method is also illustrated on the parametric study of an aircraft wing under uncertainty.

Keywords: Random field discretization; Adaptive design of experiments; Polynomial chaos expansion; Kriging

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1. Introduction

This study addresses the characterization of extreme value statistics of continuous second order random fields. It is motivated by the analysis of parametric engineering problems under parametric uncertainty. Indeed, let $y = \mathcal{M}(\boldsymbol{x}, \boldsymbol{d})$ be an engineering model, where $\mathcal{M}$ is a deterministic solver, $\boldsymbol{d} \in \mathbb{R}^{n_d}$ is a vector of control variables of the model and $\boldsymbol{x}$ is a vector of n model parameters defined on a parametric space $\boldsymbol{S} \subset \mathbb{R}^n$. The minimum (or maximum) value of $y(\boldsymbol{x}, \boldsymbol{d})$, as well as its position in the parametric space $\boldsymbol{S}$, are generally of interest in the study of the model y (*e.g.* for optimization of the modeled system).

It is now assumed that parameters $\boldsymbol{x}$ and $\boldsymbol{d}$ are affected by uncertainty. In order to take into account this uncertainty, the probabilistic framework is used. Hence, a probability space (Ω, F, P) and a random vector $\boldsymbol{\xi} : (\Omega, F) \to (\mathbb{R}^{n_s}, \mathcal{B}_{n_s})$, where $\mathcal{B}_{n_s}$ is the Borel σ algebra and n_s the stochastic dimension of the problem, are introduced. The parameters of the problem are expressed by the random variables $\boldsymbol{X}(\boldsymbol{x}, \boldsymbol{\xi}) = f(\boldsymbol{x}, \boldsymbol{\xi})$ and $\boldsymbol{D}(\boldsymbol{d}, \boldsymbol{\xi}) = g(\boldsymbol{d}, \boldsymbol{\xi})$, where $f : \mathbb{R}^n \times \mathbb{R}^{n_s} \to \mathbb{R}^n$ and $g : \mathbb{R}^{n_d} \times \mathbb{R}^{n_s} \to \mathbb{R}^{n_d}$ are the known mappings that describe how the uncertainty affects the parameters $\boldsymbol{x}$ and $\boldsymbol{d}$ respectively. The parametric random problem, $Y(\boldsymbol{X}(\boldsymbol{x}, \boldsymbol{\xi}), \boldsymbol{D}(\boldsymbol{d}, \boldsymbol{\xi})) = \mathcal{M}(\boldsymbol{X}(\boldsymbol{x}, \boldsymbol{\xi}), \boldsymbol{D}(\boldsymbol{d}, \boldsymbol{\xi}))$ can thus be modeled as a scalar random field over the parametric space $\boldsymbol{S}$. As the purpose of this article is the study of this random field with respect to the variables $\boldsymbol{x}$ and the random variables $\boldsymbol{\xi}$, the problem can be rewritten, without any loss of generality, as $Y(\boldsymbol{X}(\boldsymbol{x}, \boldsymbol{\xi}), \boldsymbol{D}(\boldsymbol{d}, \boldsymbol{\xi})) = Y(\boldsymbol{x}, \boldsymbol{\xi})$ in order to lighten the notations.

In the following we are interested in the random variable modeling the minimum (or the maximum) of the random field $Y(\boldsymbol{x}, \boldsymbol{\xi})$, denoted by

$$Y_{min}(\boldsymbol{\xi}) = \min_{\boldsymbol{x} \in \boldsymbol{S}} \left(Y(\boldsymbol{x}, \boldsymbol{\xi})\right), \tag{1}$$

as well as the random variable modeling the position where this minimum (or maximum) is reached,

$$X^{\star}(\boldsymbol{\xi}) = \arg \min_{\boldsymbol{x} \in \boldsymbol{S}} \left(Y(\boldsymbol{x}, \boldsymbol{\xi})\right). \tag{2}$$

As they allow to characterize the dispersion of the value and the position of the minimum of the random parametric problem under study, the random variables $Y_{min}(\boldsymbol{\xi})$ and $X^{\star}(\boldsymbol{\xi})$ are important information for many applications in the domains of optimization, robustness and reliability. Note that the approach we will propose can handle both cases (maximum or minimum) of extreme values of the considered random field (the passage from one to the other being done by taking the opposite of the random field). In order to simplify notations we will from now on only consider the minimization case.

The study of extreme values generally deals with a discrete collection of independent random variables. In this particular case theoretical results are available for modeling the probability distribution of the maxima (e.g. the generalized extreme value distribution, see [1]). Concerning the study of continuous random process and fields, readers are referred to [2] for theoretical basis. Once again, some theoretical results are available for some particular random process and fields (see [2] for details). Otherwise, one can rely on numerical approximations, for example, [3] proposes to use an approximation of the random field by Karhunen Loève (KL) decomposition and Gaussian mixture in order to compute the extreme value statistics by application of the Rice formula [4] on this approximation.

The approach proposed in this paper is complementary to these previous approaches, by addressing a typical industrial context in which the deterministic model $\mathcal{M}(\boldsymbol{x}, \boldsymbol{\xi})$ can only be sampled at a given space position $\boldsymbol{x}^{(0)} \in \boldsymbol{S}$ and for a given random realization $\boldsymbol{\xi}^{(0)} \in \mathbb{R}^{n_s}$ (this type of application is sometimes referred as *black box* problem). Moreover, it is assumed that the response of this deterministic model is the output of a computationally expensive numerical simulation making the use of brute force Monte Carlo approach intractable. In this context, uncertainty quantification (at a given space point $\boldsymbol{x}^{(0)}$) by polynomial chaos expansion (PCE) is now well established and will also be used in the present work. Indeed several studies have demonstrated the effectiveness of PCE to deal with uncertainty quantification, from the pioneering work by Ghanem and Spanos [5] on stochastic finite elements by Hermite PCE to the generalized PCE [6,7] and adaptive PCE [8,9]. Computation of PCE approximations also made great progress in the last decade, in particular with the development of non intrusive approaches [10–14] that allow to compute PCE only by sampling the deterministic solver $\mathcal{M}$. These developments make PCE easy to implement even on heavy computational solvers $\mathcal{M}$ (e.g. non-linear finite elements simulations).

The study of extreme values is obviously closely related to optimization. In the context of solving parametrized optimization problems, an increasingly popular approach is surrogate based optimization [15–17], where a surrogate model [18,19] is constructed in order to aid the optimization process. Kriging based optimization algorithms have