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h–p adaptive model based approximation of moment free sensitivity indices

Tanmoy Chatterjee*, Rajib Chowdhury

Department of Civil Engineering, Indian Institute of Technology Roorkee, Roorkee 247667, India

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Abstract

Moment free sensitivity analysis computes importance of input parameters by taking into account the entire probability distribution of the output response. Due to improvement in the framework of moment free sensitivity analysis, it is widely preferred over other approaches. However, the framework often becomes computationally intensive especially in large-scale systems involving finite-element simulation. In order to minimize the computational effort, two h–p adaptive meta-models have been proposed for replacing the expensive actual response evaluations. The proposed meta-models incorporate a three-layered advantage over the conventional ones, which are, global refinement of the basis functions, enabled with compressive sampling based methods based on ℓ_0- and ℓ_1-norms (p-adaptivity) and integrated with an optimal sequential experimental design scheme (h-adaptivity). Both of the proposed models have been merged efficiently in the framework of moment free sensitivity analysis. Five numerical examples have been carried out for accessing the performance of the proposed models. Finally, a real-time engineering structure has been modelled and sensitivity analysis is performed. Significant reduction in computational effort has been achieved along with good level of accuracy by utilizing the proposed meta-model assisted sensitivity tools. The results have been validated with that of Monte Carlo simulation.

Keywords: Compressive sampling; Kriging; ANOVA; h–p adaptive; Quadrature; SPGL1

1. Introduction

Uncertainties are congenital in physical processes and hence, considering them during simulation and modelling of the actual concerned phenomenon becomes crucial [1]. The sole motive is to scrutinize the effect of randomness in input parameters of a system on the response quantities of interest, also referred to as uncertainty quantification (UQ). In this context, evaluation of relative importance of a particular input variable in determining the output response is integral for understanding the stochastic mechanics and related system design. It is commonly known as sensitivity analysis (SA) and forms an imperative fragment of UQ [2]. SA being a prominent segment in stochastic computations, has found wide end applications [3–7].

* Corresponding author.
E-mail addresses: tchat.dce2014@iitr.ac.in (T. Chatterjee), rajibfce@iitr.ac.in (R. Chowdhury).

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After two decades of research in SA, the domain has been well-explored and presently positioned in quite an advanced stage. Out of the existing tools to carry out SA, the significant ones have been discussed henceforth. One of the most straightforward SA techniques are the differential methods [8]. In these methods, sensitivity of an input variable is computed by differentiating the output with respect to that variable. Generally, differentiation is either performed directly or by employing finite-difference. The drawback associated with direct differentiation is that it becomes obsolete in absence of explicit input–output relationship. Whereas, a major problem encountered while utilizing finite-difference is that the stochastic framework becomes computationally prohibitive, especially in case of expensive finite-element (FE) analysis. Other such instances of local SA include, perturbation technique [9] and score function method [10].

Global sensitivity analysis (GSA) tools [2,11,12] are more favourable for carrying out SA as compared to the above class of approaches. It is due to the fact that in local SA techniques, the individual effect of the parameters computed at limited points is not thoroughly realized in stochastic analysis. A popular class of approaches for GSA, referred to as the variance based methods, guarantee all the three adequate requirements, namely, "*global, quantitative and model free*" as underlined in [13]. More specifically, the basis of such merits lie in taking into account the overall support range of the input variables, identification of individual contribution and model independence. Extensive investigation on the theoretical and numerical analysis of variance based methods can be found in the works of Saltelli [13], Sobol [11,14], and Rabitz [15,16].

However, variance based methods as the name suggest, primarily depend on the variance of the output response. The same fact is highlighted in the words of Saltelli [13] as, variance based methods "*implicitly assume that this moment is sufficient to describe output variability*", which is entirely unsatisfactory. Thus, a sensitivity index should correspond to an entire probability distribution of the output, rather than its moment. In this context, a fourth goodness indicator to a GSA tool (first three discussed above) has been incorporated as moment independence by Borgonovo [17]. This class of GSA approaches are referred to as moment free sensitivity analysis tools.

Since moment free sensitivity analysis alleviates all the above drawbacks of SA and has been observed to yield authentic results, it has been utilized in the present study. However, this framework entails high computational effort and suffers tremendously in solving moderate to high dimensional problems involving implicit response functions (as in FE models) [18]. In order to address this issue and minimize the computational burden, two efficient novel meta-modelling tools have been integrated in the framework of moment free sensitivity analysis. Both the proposed tools are based on Kriging and incorporate the following improvements:

- The approximation potential of existing Kriging model has been enhanced by global refinement of its basis functions.
- Computational effort has been reduced by utilizing efficient screening methods based on ℓ_0- and ℓ_1-norms, which induce p-adaptive sparse configuration.
- Further computational effort has been minimized by employing an effective scheme of sequential experimental design, which imparts h-adaptivity in the models.

The rest of the paper has been organized in the following sequence. Section 2 explains the fundamentals of moment free sensitivity analysis. The proposed meta-modelling tools have been illustrated in Section 3. In Section 4, the proposed efficient moment free sensitivity analysis framework has been explained. Numerical study has been carried out in Section 5. In Section 6, a real-time engineering application has been presented. Finally, the study has been summarized in Section 7.

2. Moment free sensitivity analysis

According to classical utility theory as pointed out in [19], variance is acceptable to be representative of the complete probability distribution under the following two circumstances:

- Utility function is of quadratic nature,
- The stochastic parameter follows normal distribution.

In this context, a moment free index (MFI) for GSA has been proposed in [20]. It can be expressed as,

$$\mathrm{MFI}_i = \frac{\left\{\int \left(g_z^i - g_z^0\right)^2 \mathrm{d}z\right\}^{1/2}}{E\left(G^0\right)} \tag{1}$$