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An efficient isogeometric solid-shell formulation for geometrically nonlinear analysis of elastic shells

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Abstract

In this work an isogeometric solid-shell model for geometrically nonlinear analyses is proposed. It is based on a linear interpolation through the thickness and a NURBS interpolation on the middle surface of the shell for both the geometry and the displacement field. The Green–Lagrange strains are linearized along the thickness direction and a modified generalized constitutive matrix is adopted to easily eliminate thickness locking without introducing any additional unknowns and to model multi-layered composite shells. Reduced integration schemes, which take into account the high continuity of the shape functions, are investigated to avoid interpolation locking and to increase the computational efficiency. The relaxation of the constitutive equations at each integration point is adopted in the iterative scheme in order to reconstruct the equilibrium path using large steps and a low number of iterations, even for very slender structures. This strategy makes it possible to minimize the number of stiffness matrix evaluations and decompositions and it turns out to be particularly convenient in isogeometric analyses.

Keywords: Geometric nonlinearities; Isogeometric analysis; Composite shells; Reduced integration; MIP Newton

1. Introduction

In recent years an increasing amount of research has aimed at developing new efficient solid-shell finite elements (FEs) [1–5] for nonlinear analysis of thin structures. This is due to the advantages of these kinds of elements in comparison to classical shell ones. In particular, they allow the use of 3D continuum strain measures employing translational degrees of freedom only, so avoiding complex and expensive rules for updating the rotations. Solid-shell elements are often based on a linear displacement interpolation in order to achieve computational efficiency and then exhibit shear locking, also present in traditional shell elements, and trapezoidal and thickness locking, typical of solid elements [6]. These kinds of locking are usually sanitized by means of Assumed Natural Strain (ANS), Enhanced Assumed Strain (EAS) [7,8] and mixed (hybrid) formulations [1,9,10]. Solid-shells have been used to

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model composites or laminated beams [3,9,11] and shell structures in both the linear [4,7,12] and nonlinear [1,2,8] range. Among the most effective and interesting proposals are the mixed solid-shell elements of Sze and co-authors [1] which extend the initial PT18β hybrid element of Pian and Tong [13] to thin shells and eliminate thickness locking by means of a modified generalized constitutive matrix. This approach makes it possible, as opposed to EAS, to avoid the introduction of additional degrees of freedom (DOFs) and to obtain good predictions for multi-layered composites. Although there is the effective elimination of the interpolation locking, low order solid-shell elements exhibit a poor behavior when analyzing curved geometries. High order Lagrangian interpolations, on the other hand, have been little used due to the high number of DOFs and computational cost for the integration and assembly of the quantities [14].

The isogeometric analysis (IGA) [15,16] represents a good alternative to high order Lagrangian FEs. The main reason for its success is, in our opinion, the way it makes it possible to elevate the order of the shape functions while practically maintaining the same number of DOFs of linear Lagrangian interpolations. Another notable feature is that the high order continuity of the shape functions allows the total number of integration points to be reduced significantly as shown in [17,18] compensating for the computational cost of the assembly of the discrete operators. Finally, the geometry is reproduced exactly, regardless of the mesh adopted and a simple link between CAD and structural analysis is available.

These considerations make IGA very attractive, particularly in geometrically nonlinear analysis where a highly continuous solution is often expected [19–21]. However, there are some difficulties associated with IGA with respect to traditional finite elements. The use of very high order shape functions eliminates interpolation locking but, at the same time, increases the computational effort for the integration and the assembly of the discrete quantities and for the solution of the discrete problem because of the decrease in the stiffness matrix sparsity. For these reasons C^1 and C^2 NURBS interpolations are often preferred, even if they are not immune to locking phenomena. Due to the inter-element continuity of the interpolation, element-wise reduced integrations and strategies, like ANS [22], only alleviate, but do not eliminate locking, and so are not effective for very thin shells. For the same reason, mixed formulations with stress shape functions defined at element level are not able to prevent locking. Conversely, mixed formulations with continuous stress shape functions have been successfully proposed [23,24]. However, in this way the total number of DOFs increases with respect to the initial displacement formulation and the static condensation of the stress variables, usually employed in FE analysis and performed at the element level, can be carried out only at patch level and as a result is not convenient because it produces a full condensed stiffness matrix. An interesting alternative is the use of displacement formulations with patch-wise reduced integration rules [17]. These have been shown to alleviate and, in some cases, eliminate interpolation locking in linear elastic problems [18] employing a low number of integration points and so significantly improve the computational efficiency. This strategy seems more attractive than the mixed formulation, since it preserves the stiffness matrix sparsity without introducing additional unknowns and allows a more efficient integration.

However, when comparing mixed and displacement formulations in path-following geometrically nonlinear analyses, many authors observed that the mixed ones can withstand much larger step sizes (increments) with a reduced number of iterations to obtain an equilibrium point and then the equilibrium path. The reason for this is explained in [25,26] where it is shown that the performances of the Newton method drastically deteriorate in displacement formulations when the slenderness of the structure increases. Conversely, the Newton method in mixed formulations is unaffected by this phenomenon, which depends only on the format of the iterative scheme adopted (mixed or purely displacement based) and also holds when a mixed and a displacement formulation provide the same discrete accuracy. To eliminate this inconvenience in displacement-based finite elements the Mixed Integration Point (MIP) strategy has been recently proposed in [27]. It consists of the relaxation of the constitutive equations at each integration point during the Newton iterative process.

In this work, we propose an isogeometric solid-shell formulation for geometrically nonlinear analyses of homogeneous and composite multi-layered shells, which uses a linear through-the-thickness interpolation of geometry and displacements. The nonlinear model is based on a Total-Lagrangian formulation adopting the Green–Lagrange strain measure. A linearization of the strains and a pre-integration along the thickness direction allow the definition of a modified generalized constitutive matrix, which effectively eliminates thickness locking without introducing any additional through-the-thickness DOF [28] and leads to accurate predictions for composites. The displacement field and the geometry are rewritten in terms of semi-sum and semi-difference of the top and bottom surface quantities. The model so obtained allows a bidimensional description and interpolation of the geometry and displacements using 2D NURBS of generic order and continuity. Each control point is equipped with six DOFs but, in contrast