Available online at www.sciencedirect.com
ScienceDirect
Comput. Methods Appl. Mech. Engrg. 331 (2018) 184–196
Computer methods in applied mechanics and engineering
www.elsevier.com/locate/cma

Data-based derivation of material response

Adrien Leygue*, Michel Coret, Julien Réthoré, Laurent Stainier, Erwan Verron

GeM - Research Institute of Civil Engineering and Mechanics, UMR 6183, CNRS - Ecole Centrale de Nantes, Université de Nantes, 1, rue de la Noé, 44321 Nantes, France

Received 10 May 2017; received in revised form 10 October 2017; accepted 8 November 2017
Available online 21 November 2017

Abstract

This paper proposes a method to identify the strain–stress relation of non-linear elastic materials, without any underlying constitutive equation. The approach is based on the concept of Data Driven Computational Mechanics recently introduced by Kirchdoerfer and Ortiz (2016). From a collection of non-homogeneous strain fields, for example measured through Digital Image Correlation, the algorithm builds a database of strain–stress couples that sample the mechanical response of the material for the range of measured strains. The method is first derived for truss structures and then extended to the case of small-strain elasticity. The method accuracy, sensitivity to measurement noise and parameters are discussed using manufactured data.

Keywords: Data Driven Computational Mechanics; Digital Image Correlation; Constitutive equation; Material response; Machine learning

1. Introduction

Constitutive equations constitute a key concept in mechanical engineering as they relate strain and stress for a given material. The parameters of a constitutive equation are usually adjusted considering a sufficient set of experimental data and appropriate fitting procedures. Beyond the mere description of the mechanical response, a constitutive equation has several purposes:

- it provides a smooth strain–stress relation in which experimental noise has been smeared out,
- for given loading conditions, *e.g.* uniaxial extension, it both interpolates between individual measurements and extrapolates the material response,
- its tensorial nature naturally extends the stress–strain relation to multiaxial loading conditions that are difficult to attain experimentally.

Recently, Kirchdoerfer and Ortiz [1,2] have introduced the concept of Data-Driven Computational Mechanics (DDCM in the following) for elastic materials, in which the constitutive equation vanishes and is replaced by

* Corresponding author.
E-mail address: adrien.leygue@ec-nantes.fr (A. Leygue).

https://doi.org/10.1016/j.cma.2017.11.013

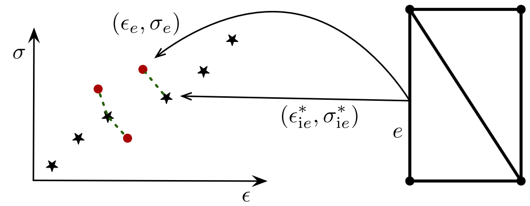

Fig. 1. The two states (ϵ_e, σ_e) and $(\epsilon^*_{ie}, \sigma^*_{ie})$ associated to a truss element e. The dashed line represents the energetic mismatch between the two states. On the left we see that the mapping ie between elements and material states can assign the same particular material state to two different elements.

a database of strain–stress couples (called states) which sample the mechanical response of the material. In this approach, regularization/smoothing/interpolation of experimental data are carried out during the computation of the numerical solution of the boundary value problem. The results presented by the authors are encouraging and open the door to many perspectives from the modeling point of view, since the necessity of an explicit or implicit strain–stress relationship is relaxed. In the wake of these contributions, Ibañez et al. [3] investigate the possible use of manifold learning techniques applied to the material database.

Let us briefly recall the method of Kirchdoerfer and Ortiz for data-driven simulation in the particular case of truss structures [1]. It seeks to assign to each truss element e a mechanical state and a material state, a state being a strain–stress couple. Considering both mechanical equilibrium (involving stress) and compatibility conditions (involving strain) as non-questionable, the mechanical state of element e consists of a strain–stress pair (ϵ_e, σ_e) that exactly satisfies the above equations which can be considered as constraints. The second state associated to e, denoted $(\epsilon^*_{ie}, \sigma^*_{ie})$, is called the material state and is extracted from a collection of admissible material states for the material: $(\epsilon^*_i, \sigma^*_i)$, where $i \in 1 : N^*$. The index i$e \in 1 : N^*$ specifies the material state of element e. It should be interpreted as a pointer that assigns to each element a specific material state in the material database. Fig. 1 illustrates the main ideas behind the method. The proposed solver seeks, for every element simultaneously, a mechanical and a material state as close to each other as possible and such that the former satisfies mechanical equilibrium and compatibility conditions. This is formally expressed as follows:

$$\text{solution} = \arg \min_{\epsilon_e, \sigma_e, \mathrm{ie}} \frac{1}{2} \sum_e w_e \|(\epsilon_e - \epsilon^*_{\mathrm{ie}}, \sigma_e - \sigma^*_{\mathrm{ie}})\|^2_C \,, \tag{1}$$

subject to

$$\sum_e w_e \mathbf{B}_{ej} \sigma_e = \mathbf{f}_j \,, \tag{2}$$

and

$$\epsilon_e = \sum_j \mathbf{B}_{ej} \mathbf{u}_j \,. \tag{3}$$

In the above equations, $\|(\epsilon, \sigma)\|_C$ is an energetic norm, the matrix $\mathbf{B}_{ej}$ encodes the connectivity and geometry of the truss, and w_e denotes the volume of the truss element e. Furthermore, $\mathbf{u}_j$ and $\mathbf{f}_j$ represent, respectively, the displacement and the force applied to truss nodes. For the particular choice

$$\|(\epsilon_e, \sigma_e)\|^2_C = (C\epsilon_e^2 + \frac{1}{C}\sigma_e^2) \,, \tag{4}$$

the authors propose an efficient algorithm to solve this problem of combinatorial complexity. The constant parameter (possibly defined element-wise) C is the only parameter of the method and can be interpreted as a modulus associated to the mismatch of mechanical and material states. This mismatch is represented by the dashed lines in Fig. 1.

Although there is no need for a constitutive equation, the database of material states is a mandatory pre-requisite of the method before starting the simulation. Building this database computationally, for example through micro–macro approaches such as FE^2 [4,5], is expensive and might require efficient model order reduction and high dimensional interpolation techniques. From an experimental point of view however it is far from trivial to be able to, somehow,