Accepted Manuscript

Model order reduction of nonlinear homogenization problems using a Hashin-Shtrikman type finite element method

Stephan Wulfinghoff, Fabiola Cavaliere, Stefanie Reese

PII: S0045-7825(17)30690-4
DOI: https://doi.org/10.1016/j.cma.2017.10.019
Reference: CMA 11647

To appear in: *Comput. Methods Appl. Mech. Engrg.*

Received date : 11 July 2017
Revised date : 8 October 2017
Accepted date : 20 October 2017

Model Order Reduction of Nonlinear Homogenization Problems Using a Hashin-Shtrikman Type Finite Element Method

Stephan Wulfinghoff[a], Fabiola Cavaliere[a], Stefanie Reese[a]

[a]*Institute of Applied Mechanics, RWTH Aachen University, Mies-van-der-Rohe-Str. 1, 52074 Aachen, Germany*

{*stephan.wulfinghoff, fabiola.cavaliere, stefanie.reese*}*@rwth-aachen.de*

Abstract

This work presents a computational nonlinear homogenization approach, the starting point of which is a model order reduction method based on data-clustering. To this end, the micromechanical data from numerical experiments (snapshots) is analyzed in order to identify characteristic microstructural deformation patterns. These describe how the macroscopic strain typically localizes within the microstructure. The outcome of the procedure is a subdivision of the microstructure into a set of clusters of material points. Within each cluster the strain is then approximated as being constant.

The mechanical problem is formulated in terms of a three-field Hashin-Shtrikman type variational formulation which is based on the introduction of a linear-elastic reference medium. After discretization, most of the global unknowns can be eliminated via static condensation leaving the piecewise constant cluster strains as the primary unknowns.

The resulting homogenization scheme includes, as special cases, the finite element method as well as Hashin-Shtrikman and Talbot-Willis type homogenization approaches with phase-wise constant trial fields (as well as related bounds). The limit case 'finite element method' allows to transfer knowledge from finite element technology and thus provides new strategies for the choice of the stiffness of the reference material. The method is applied to several nonlinear microstructures with different inclusion volume fractions and varying degree of anisotropy. The results are shown to be in good agreement with full-field FE-simulations. Furthermore, the method is used to compute a refined upper bound of the Talbot-Willis type (compared to phase-wise constant trial fields), which converges to the finite element solution with increasingly refined discretization.

Keywords: homogenization, model order reduction, finite element technology, variational formulation, Hashin-Shtrikman

1 Introduction

Engineering materials are typically heterogeneous on the microscale. The features of the microstructure, like the size, the spatial distribution and the orientation of the constituents determine the macroscopic material behavior (e.g., Segurado and Llorca, 2006; Wulfinghoff and Böhlke, 2012; Böhlke and Lobos, 2014). There is a huge body of literature on the mechanics of heterogeneous materials. Typical examples include investigations of metallic materials with hard inclusions embedded in a soft matrix (e.g., Wulfinghoff and Böhlke, 2015) or carbon-fibres in a polymer-matrix (e.g., Stier et al., 2015).

There is a growing scientific and economic need to take the material's microstructure into account, and the interest in continuum mechanical two-scale models of engineering systems is increasing. Such models typically consist of a macroscopic model, being defined on the structural scale, which is coupled to a microscopic model of the material microstructure. The latter