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Risk-averse structural topology optimization under random fields using stochastic expansion methods

Jesús Martínez-Frutos[a,*], David Herrero-Pérez[a], Mathieu Kessler[b], Francisco Periago[a,b]

[a] *Computational Mechanics and Scientific Computing Group, Technical University of Cartagena, Campus Muralla del Mar, 30202 Cartagena, Murcia, Spain*

[b] *Department of Applied Mathematics and Statistics, Technical University of Cartagena, Campus Muralla del Mar, 30202 Cartagena, Murcia, Spain*

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Highlights

- Risk-averse topology optimization considering random field loading and material uncertainty is proposed.
- The existence of a solution for the risk-averse topology optimization problem is proved.
- Stochastic shape derivatives of continuous structures are obtained.
- An adaptive anisotropic polynomial chaos approach is used to approximate the functional cost and its sensitivity.
- Similarities between robust and risk-averse optimal designs are discussed.

Abstract

This work proposes a level-set based approach for solving risk-averse structural topology optimization problems considering random field loading and material uncertainty. The use of random fields increases the dimensionality of the stochastic domain, which poses several computational challenges related to the minimization of the Excess Probability as a measure of risk awareness. This problem is addressed both from the theoretical and numerical viewpoints. First, an existence result under a typical geometrical constraint on the set of admissible shapes is proved. Second, a level-set continuous approach to find the numerical solution of the problem is proposed. Since the considered cost functional has a discontinuous integrand, the numerical approximation of the functional and its sensitivity combine an adaptive anisotropic Polynomial Chaos (PC) approach with a Monte-Carlo (MC) sampling method for uncertainty propagation. Furthermore, to address the increment of dimensionality induced by the random field, an anisotropic sparse grid stochastic collocation method is used for the efficient computation of the PC coefficients. A key point is that the non-intrusive nature of such an approach facilitates the use of High Performance Computing (HPC) to alleviate the

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[*] Corresponding author.

E-mail addresses: jesus.martinez@upct.es (J. Martínez-Frutos), david.herrero@upct.es (D. Herrero-Pérez), mathieu.kessler@upct.es (M. Kessler), f.periago@upct.es (F. Periago).

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computational burden of the problem. Several numerical experiments including random field loading and material uncertainty are presented to show the feasibility of the proposal.

Keywords: Risk-averse topology optimization; Random fields; Material uncertainty; Stochastic expansion methods; Existence theory; Level-set method

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1. Introduction

Topology optimization aims at finding the optimal distribution of material within a design domain such that an objective functional is minimized under certain constraints [1]. Contrary to size and shape optimization methods, topology optimization permits to obtain a material distribution without assuming any prior structural configuration. This provides engineering designers with a powerful tool to find innovative and high-performance conceptual designs at the early stages of the design process. Such a technique has been successfully applied to improve the design of complex industrial problems, such as aeronautical, aerospace and naval applications [2]. However, deterministic conditions are usually assumed, and thus obviating the different sources of uncertainty which may affect not only the safety and reliability of structures but also their performance. Robustness and reliability have been recognized as a desirable property in structural systems and have sparked considerable interest in the scientific and engineering communities. The need for including uncertainty quantification and propagation stages during the design process has shown to be a key issue for solving real-world engineering problems in several fields, such as aeronautical and aerospace [3], civil [4], automotive [5] and mechanical [6] engineering, to name but a few. This fact, together with the development of probabilistic uncertainty propagation methods, has fostered the interest for considering uncertainty within the topology optimization problems, giving rise to the formulation of several approaches embraced under the term of Topology Optimization Under Uncertainty (TOUU) methods.

Such methods can be broadly classified, according to the representation and treatment of uncertainties, into non-probabilistic and probabilistic approaches. The former methods [7] do not require the statistical information about the uncertainty of the phenomenon but a qualitative notion about its magnitude. The worst-case approach [8–10], taking the form of a min–max optimization problem, and fuzzy techniques [11], making use of fuzzy set theory, are methods included in this category. The main drawback of these approaches is that they are often too conservative, due to overestimation of uncertainty, and may lead to optimal designs with poor structural performance. The latter methods make use of the probabilistic characterization of the uncertainty of the phenomenon. Several formulations have been proposed in this context to address the wide concept of "structural robustness". These formulations differ from each other in the design objective as well as in the way the uncertainty is incorporated within the optimization formulation. Robust Topology Optimization (RTO) incorporates the first two statistical moments of the cost functional to obtain optimal designs which are less sensitive to variations in the input data [12–16]. Reliability-Based Topology Optimization (RBTO) aims at determining the best design solution (with respect to prescribed criteria such as stiffness, weight or construction costs) while explicitly considering the unavoidable effects of uncertainty. This is done by posing the constraints in terms of the probability of constraint violation (probability of failure) [17–19]. On the other hand, Risk-Averse Topology Optimization (RATO) [20–22] does not aims at minimizing a deterministic prescribed criteria but a risk function that quantifies the expected loss related with the damages (e.g. excess probability) [23]. Whereas RBTO provides optimal designs in terms of a deterministic prescribed criteria with enough reliability level, RATO provides the best design from the point of view of risk-aversion.

One of the main challenges of TOUU methods is the computational burden of addressing the problem, which still remains even though significant numerical and theoretical advances have been achieved in the last years. A key point of the computational challenge is that when the solution of the underlying Partial Differential Equation (PDE) is expensive, one can only afford to solve a few hundred samples. This is far from the required number of samples for estimating a probability. Such a drawback is exacerbated in high-dimensional stochastic domains, such as those obtained with random fields. This work is concerned with this issue in the context of RATO problems under random fields.

The RATO problem is introduced by the early work of Conti et al. [21] proposing a risk-averse approach in the finite dimensional context of shape optimization. The uncertainty is introduced in the applied loads, and hence, there