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Absorbing boundary condition for scalar-wave propagation problems in infinite media based on a root-finding algorithm

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Highlights

- New boundary conditions are proposed for unbounded domains to absorb scalar waves.
- A root-finding algorithm for the solution of exact dispersion relation is utilized.
- In the present study, we select the Newton–Raphson method as the algorithm.
- We assess the accuracy of the new boundary condition and verify its stability.
- We demonstrate that the proposed approach leads to accurate and stable computations.

Abstract

In this study, we propose a new approach for development of absorbing boundary conditions for scalar-wave propagation problems in infinite media based on a root-finding algorithm for the solution of the exact wave dispersion relation. We select the Newton–Raphson method as the root-finding algorithm in the present study and assess the accuracy of the newly developed boundary condition by estimating its reflection coefficient. Furthermore, we evaluate and verify the stability of the boundary condition. We apply our development to various scalar-wave propagation problems and demonstrate that the proposed approach leads to accurate and stable computations.

Keywords: Root-finding absorbing boundary condition (RFABC); Absorbing boundary condition; Wave propagation; Scalar wave; Infinite medium; Root-finding algorithm

1. Introduction

The theory of wave propagation is the basis of various physical phenomena in fields such as civil engineering, mechanical engineering, offshore engineering, seismology, meteorology, and oceanography [1]. In many applications, we encounter wave propagation phenomena when dealing with unbounded or infinite media such as semi-infinite

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layered ground, ocean, atmosphere, etc., rather than a finite, bounded medium. Such applications require that we simulate the dynamic behaviors of infinite media accurately and efficiently.

Wave propagation phenomena in infinite media can be simulated using computational treatments such as those based on the finite element method and the finite difference method. However, since these treatments were developed originally for problems in finite domains, it is necessary to use a special numerical or mechanical model that can precisely consider the energy radiation into infinity. Otherwise, the energy will be trapped in a finite region, resulting in very different results. Therefore, various models such as consistent transmitting boundaries [2], boundary elements [3,4], infinite elements [5], high-order non-reflecting boundary conditions (NRBCs) or absorbing boundary conditions (ABCs) [6], and perfectly matched layers (PMLs) [7,8] have been developed and used for various wave propagation problems in infinite media.

Various nonlinear behaviors, for example, material nonlinearities, may occur in wave propagation phenomena. We can address these most conveniently in the *time* domain. Therefore, with infinite media, it is best to consider energy radiation into infinity directly in the time domain. Accuracy and efficiency depend on how well we can account for energy radiation into infinity in the time domain. Among the models mentioned above that can consider the influence of the infinite domain, the higher-order ABCs and PMLs can guarantee both accuracy and efficiency in the time domain and have been applied widely to wave propagation problems in the time domain [9].

The higher-order ABCs can be implemented to any desired degree of accuracy using auxiliary variables. However, since the auxiliary variables are virtual ones that exist only on the boundary of an infinite region, it is desirable that the higher-order ABCs do not include any derivatives of the auxiliary variables in the direction perpendicular to the boundary. In other words, the high-order ABCs should include only derivatives in time and spatial direction parallel to the boundary. This can lead to greater restriction in the development and implementation of new high-order ABCs. Thus, recent studies have proceeded to develop boundary conditions that can address this issue [6,10–12]. On the other hand, PMLs have an advantage because of easy implementation in existing numerical frameworks such as the finite element method and the finite difference method because of the relatively straightforward complex-coordinate transformation.

The higher-order ABCs converge to exact solutions and their accuracy increases with the order. In addition, necessary and sufficient conditions for the boundary conditions to produce stable solutions of wave propagation problems are available [13,14]. On the other hand, it was observed that long-term computations using conventional PMLs can lead to unstable solutions for wave propagation problems. In order to resolve the instability, a multiaxial perfectly matched layer (M-PML) was proposed and its accuracy and stability were studied [15,16]. The stability of the M-PML was proved and it was shown that stable absorbing boundaries can be constructed with the model.

Among the available models, the higher-order ABCs and PMLs for time-domain applications entail the advantages and disadvantages referred to above. In this study, we propose a new absorbing boundary condition, accurate and efficient for scalar-wave propagation problems in infinite media, without the disadvantages of available models. We have developed our boundary condition using a new approach. Specifically, while the high-order ABCs approximate the dispersion equation of waves in the infinite region by rational expressions or a series of simple differential operators and the PMLs introduce artificial damping through complex transformations of the spatial coordinate system, in this study, a solution of the dispersion equation is obtained using the Newton–Raphson method. The solution does *not* require any derivatives of auxiliary variables in the direction perpendicular to the boundary of the infinite region and the disadvantage of high-order ABCs *is eliminated*. Furthermore, we show that our development satisfies necessary and sufficient conditions for stable solutions to wave propagation problems.

The new boundary condition is formulated and its accuracy and stability are analyzed in Sections 2.1 and 2.2. We present a finite-element formulation of the boundary condition in Section 2.3. In Section 3, we apply the newly developed boundary condition to various scalar-wave propagation problems in wave-guides in order to examine its accuracy and stability. In order to apply this kind of boundary conditions to wave-propagation problems in a full- or a half-space, boundary conditions for corner regions must be formulated [17–19]. Since the primary purpose of this study is the development and verification of an absorbing boundary condition based on a new approach, the subject of this research is limited only to problems in wave-guides which are considered sufficient for the purpose. The development of the corner-compatible boundary condition will be a topic in a future study. Finally, we summarize our study in Section 4.