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An unusual amplitude growth property and its remedy for structure-dependent integration methods

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Highlights

- An unusual amplitude growth property is found for a general structure-dependent integration method.
- This amplitude growth occurs in the steady-state response of a high frequency mode.
- A local truncation error can be used to reveal the cause of this amplitude growth.
- A remedy to overcome the amplitude growth can be proposed by using the local truncation error.
- The results of this study are applicable to general structure-dependent integration methods.

Abstract

An unusual amplitude growth in the steady-state response of a high frequency mode for a structure-dependent integration method is numerically and analytically identified. This is a brand new type of amplitude growth and it has never been found in the conventional integration methods. The root cause of this unusual amplitude growth can be revealed by examining the local truncation error constructed from a forced vibration error, where the dominant error term for high frequency modes plays the key role for this unusual amplitude growth. An effective remedy is proposed by introducing a load-dependent term into the difference equation for displacement and/or velocity increment to remove the dominant error term. As a result, this adverse amplitude growth can be automatically removed. Consequently, the inclusion of the load-dependent term in the formulation of the difference equation for displacement increment and/or velocity increment is inevitable for a general structure-dependent integration method.

Keywords: Amplitude growth; Steady-state response; High frequency mode; Local truncation error; Structure-dependent integration method

1. Introduction

Many integration methods have been developed to solve the equations of motion for structural dynamics [1–3]. An integration method often consists of the difference equations for displacement and velocity increment, and the

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equation of motion or its asymptotic form. The coefficients of the two difference equations are generally some specific scalar constants for conventional integration methods, such as the well-known Newmark family method [4], HHT-α method [5], WBZ-α method [6], generalized-α method [7] and the methods developed by Zhou and Tamma [8,9]. In general, there are two basic categories of the integration methods. One is explicit [10–17] and the other is implicit [4–9,18–24]. It is recognized that for a conventional integration method an explicit method only has conditional stability and thus the step size is limited by the upper stability limit although its computation procedure for a time step is simple [1–3]. Whereas, an iteration procedure will be involved for an implicit method although there is no limitation for choosing an appropriate step size since it can have unconditional stability [1–3]. Hence, it has been shown by Dahlquist [25,26] that there is no unconditionally stable explicit method among the linear multi-step methods.

It seems that the use of matrix coefficients (i.e., structure-dependent coefficients) instead of the scalar coefficients for the difference equations may be able to overcome the Dahlquist barrier. Thus, a structure-dependent integration method with unconditional stability and explicit formulation has been first proposed by Chang in 2002 [27] and a similar integration method was developed later in 2007 [28]. These two integration methods were developed based on the assumption of using the structure-dependent matrix coefficients instead of the scalar constant coefficients for the difference equation for displacement increment. Based on the same assumption, another structure-dependent integration method was proposed by Chen and Ricles using discrete control theory [29]. These three integration methods generally have unconditional stability for linear elastic and stiffness softening systems while they will become conditionally stable for stiffness hardening systems. Hence, an improved integration method has been developed to additionally have unconditional stability for a certain stiffness hardening systems [30]. A family of structure-dependent integration methods was first developed by Chang [31], where only the difference equation for displacement increment is structure dependent. He also proposed another family method later [32], where both the difference equations for displacement and velocity increment are structure dependent. A very similar family method was developed by Gui et al. in 2014 [33]. To introduce the favorable numerical damping into a structure-dependent integration method, two families of dissipative integration methods have been successfully developed by Chang [34,35] by using the asymptotic form of the equation of motion. Similarly, another dissipative family method was developed by Kolay and Ricles later [36]. It has been found that a structure-dependent integration method can generally have unconditional stability for linear elastic and stiffness softening systems while it becomes conditional stability for stiffness hardening systems [30]. Consequently, a stability amplification factor has been proposed to overcome this difficulty and thus unconditional stability for stiffness hardening systems can be achieved [37]. These structure-dependent integration methods have been classified as either explicit or semi-explicit methods based on difference equations [38], where the former refers to the methods with explicit difference equations for both displacement and velocity while the latter refers to those with explicit difference equation for displacement only. Hence, the structure-dependent integration methods in the references [29,32,33,36] are explicit methods while the other structure-dependent integration methods are semi-explicit methods [27,28,30,31,34,35]. Notice that the semi-explicit structure-dependent integration methods might be classified as implicit methods based on Bathe's classification [39]. However, it is important to note that these integration methods, either explicit or semi-explicit, will involve no nonlinear iterations for each time step. Hence, they can integrate unconditional stability and explicit formulation together and thus they can save huge computational efforts for solving inertial problems when compared to traditional implicit methods [31,32,34,35].

An overshoot in the Wilson-θ method [40] has been discovered by Goudreau and Taylor in 1972 in the early high frequency free vibration response although it can have unconditional stability [41]. The cause of this overshoot has been explored by Hilber and Hughes and a simple technique has been proposed to detect such an overshoot [42]. Unlike the overshoot occurred in an early free vibration response, an unusual amplitude growth may experience in the steady-state response of a high frequency mode for a general structure-dependent integration method and this type of unusual amplitude growth has never been found for conventional integration methods [4–24]. Hence, the cause of this unusual amplitude growth for structure-dependent integration methods must be studied. In addition, it will be very useful if a remedy can be proposed to eliminate this unusual amplitude growth.

In a pilot study, a problem is solved to illustrate the unusual amplitude growth of the two Chang family methods [31,32] while there is no such an amplitude growth for a conventional integration method, such as the Newmark family method [4]. Next, the local truncation errors of the Newmark family method and the two Chang family methods are derived from a forced vibration response and thus the cause of this unusual amplitude growth can be revealed after the examination and comparison of these local truncation errors. In addition, a remedy is proposed