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ScienceDirect
Comput. Methods Appl. Mech. Engrg. 329 (2018) 1–23
Computer methods in applied mechanics and engineering
www.elsevier.com/locate/cma

Stress-based shape and topology optimization with the level set method

R. Picelli[a,*], S. Townsend[a], C. Brampton[b], J. Norato[c], H.A. Kim[a,d]

[a] *Cardiff School of Engineering, Cardiff University, The Queen's Buildings, 14-17 The Parade, Cardiff, CF24 3AA, United Kingdom*
[b] *Department of Mechanical Engineering, University of Bath, North Rd, Bath, BA2 7AY, United Kingdom*[1]
[c] *Department of Mechanical Engineering, University of Connecticut, United Technologies Engineering Building, Storrs, CT 06269, USA*
[d] *Structural Engineering Department, University of California, San Diego, 9500 Gilman Drive, San Diego, CA 92093, USA*

Received 8 March 2017; received in revised form 29 August 2017; accepted 1 September 2017
Available online 3 October 2017

Highlights

- Topology optimization is applied to stress-based structural design problems.
- Shape sensitivities and the level set method are used.
- Stress minimization, stress constraints and multiple load cases and stress criteria are considered.
- A p-norm function is used as stress constraints aggregation.

Abstract

This paper proposes a level set method to solve minimum stress and stress-constrained shape and topology optimization problems. The method solves a sub-optimization problem every iteration to obtain optimal boundary velocities. A p-norm stress functional is used to aggregate stresses in a single constraint. The shape sensitivity function is derived and a computational procedure based on a least squares interpolation approach is devised in order to compute sensitivities at the boundaries. Adaptive constraint scaling is used to enforce exact control of stress limits. Numerical results show that the method is able to solve the problem efficiently for single and multiple load cases obtaining solutions with smooth boundaries.

Keywords: Topology optimization; Stress minimization; Stress constraints; Level set method

1. Introduction

Stress optimization is one of the key factors for structural design in a wide range of engineering problems. Its importance relies in obtaining mechanical structures that are free of or present less stress concentrations. Topology

* Corresponding author.
E-mail address: PicelliR@cardiff.ac.uk (R. Picelli).
[1] Formerly at.

https://doi.org/10.1016/j.cma.2017.09.001

optimization has shown to be a powerful tool for structural conception. If stress is not taken into account in the concept of the structural component, the design is susceptible to postprocessing or rework, leading to unexpected costs. However, considering stress within the context of topology optimization has been an arduous task for our research field since the pioneering paper by Duysinx and Bendsøe [1].

Notably, Le et al. [2] presented a practical solution for stress-constrained design in the framework of density-based topology optimization [3]. The authors used a set of numerical ingredients to circumvent three major issues namely the singularity problem, the local nature of the stress and the highly nonlinear stress behavior. The first refers to singular optima and the difficulties that nonlinear programming algorithms have to identify degenerate subspaces where the globally optimal stress design can be located. In practice, very low density elements can present high stress values, leading to the non-removal of such elements. Stress relaxation has been applied to ensure that the element stresses and volume tend to zero simultaneously when the latter approaches zero [4–6]. The second issue occurs because stress is a local quantity, which implies that stress levels must be controlled in every material point of the structure. This leads to a large number of constraints. Global measures of the maximum stress have been used to estimate and control the stresses throughout the structure. Such measures can be based on the p-norm or Kreisselmeier–Steinhauser (K-S) functions [2,7–11]. The third related issue is that stress levels are highly sensitive to design changes. This can be specially critical in stress concentration regions, such as sharp and re-entrant corners.

The breakthrough in Le et al. [2] has been the normalization of a p-norm stress measure to approximate the peak stress in the structure, applied together with stress relaxation and consistent density filtering in a solid isotropic material with penalization (SIMP) framework. A key factor to the success of this method is that it uses a consistent density filtering versus sensitivity filtering, the latter mentioned to not perform well because of the nonlinearity of the stresses. Since then, stress-constrained topology optimization has attracted a lot of attention from researchers, including different formulations and models such as a damage approach [12], fluid–structure interaction [13] and thermo-mechanical coupling [14].

In the level set optimization framework, stress was first considered in the papers by van Miegroet and Duysinx [15] and Allaire and Jouve [16]. The first carried out shape optimization to alleviate stress concentrations in 2D fillets. The later presented stress shape derivatives and minimum stress topology solutions. One advantage of the level set method (LSM) is that the structural boundaries are well defined throughout the optimization [17].

An extensive range of techniques have been proposed throughout the last years to incorporate stress-based design into level set topology optimization methods. Most of the papers propose new penalty functions or global stress measures [18–21]. Verbart et al. [22] employed in the LSM the same set of techniques previously used in density-based methods, such as stress relaxation, p-norm aggregation function, a SIMP model and adaptive constraint scaling. Xia et al. [23,24] enhanced the performance of piezoresistive sensors by tailoring the performance of stress in regions of interest via a level set based method. Topological derivatives for stress-based functionals were also developed [25,26] and recently applied to topology design of compliant mechanisms [27]. Wang and Li [28] formulated stress-constrained topology optimization with a novel shape equilibrium problem of active constraints, removing the intrinsic nondifferentiability introduced by local stress constraints. The same idea was applied later into a stress isolation problem [29]. Activation and deactivation of constraints were also applied by Emmendoerfer and Fancello in [30] and [31], together with a few regularization techniques. The latter paper used a reaction–diffusion equation to guide the design optimization sequence in order to eliminate the level set reinitialization steps. It was shown to improve stability and convergence and new holes inside the domain were created, although the stress-constrained results did not confirm the low dependency of the initial level set function.

Most of the works mentioned above apply the Finite Element Method (FEM) or the extended FEM (XFEM) and rich discussions arise regarding the accuracy of stress computations in discretized domains. In the same scope, Cai et al. [32] and Cai and Zhang [33] combined the Finite Cell Method (FCM) with spline functions to model any type of geometry and enhance stress accuracy. Guo et al. [34] used the XFEM to develop stress related optimization with multi-phase materials. Although considerable progress was achieved, most of the works relating stress and level set topology optimization are not able to enforce stress constraints without the aid of extra numerical techniques, e.g. perimeter regularization or a compliance term added to the objective. Other papers present oscillatory boundaries and/or very thin structural members in their solutions, most of them not being able to actually remove the re-entrant corner in the benchmark L-bracket example. More recently, Polajnar et al. [35] presented good topology solutions for stress minimization and proposed a novel objective based on a penalized stress-deviation measure and approximated the level set description by the XFEM. The improvements in the level set based methods are highly significant to