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A multiscale predictor/corrector scheme for efficient elastoplastic voxel finite element analysis, with application to CT-based bone strength prediction

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Abstract

Voxel finite elements combined with plasticity have been shown to accurately predict the evolution of bone failure, but involve a prohibitive computational cost when applied to high-resolution CT scans of a complete bone. We present a simple multiscale predictor/corrector scheme that uses elasticity and the finite cell method on a coarse-scale mesh, complemented by plasticity and fine-scale voxel finite elements in regions where failure occurs. The core components of our method are top-down displacement and bottom-up stress projectors for the exchange of information between coarse and fine scales. Our choice of projectors eliminates communication of fine-scale voxel elements beyond boundaries of coarse-scale cells, which enables the solution in terms of a series of small uncoupled systems at a fraction of the computing power and memory required by the fully coupled fine-scale system. At the same time, we illustrate that the multiscale approach yields the same accuracy as the full-resolution voxel finite element method, if we appropriately balance the approximation power of coarse-scale and fine-scale meshes. We demonstrate the advantages of our method for the load capacity analysis of a patient-specific vertebra.

Keywords: Voxel finite element method; Finite cell method; Multiscale predictor/corrector scheme; Plasticity; Bone strength prediction; Parallel computing

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1. Introduction

The patient-specific prediction of bone strength has important implications for a number of clinical applications, such as osteoporosis detection and surgery planning in orthopedics, and has therefore received significant attention over the last two decades [1–7]. In this context, the finite element method (FEM) has been shown to provide very good accuracy and correlation with respect to experiments [3,5,8,9]. FEM-based methods for bone strength analysis, however, are still hardly integrated in clinical routines today due to several challenges. Standard finite element methods need a geometric model based on sharp boundary surfaces, while medical imaging technologies provide

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an explicit geometric description in the form of three-dimensional volumetric pixels (voxels), usually coupled with limited resolution and fuzzy color information. This requires time-consuming and error-prone procedures to generate analysis-suitable meshes. Moreover, bones are complex structures that consist of two major tissue types: cortical bone (the thin and stiff outer layer) and trabecular bone (more flexible foam-like inner structure). Their different mechanical properties give rise to complex fracture behavior at different scales [10–12]. Many multiscale methods [13–22] have been developed for different purposes; however, applying these methods for bone strength analysis is still difficult due to these challenges.

Voxel finite element methods [3,23–28] provide a potential pathway to overcome some of these difficulties. They associate each voxel obtained from computed tomography (CT) scans with one linear hexahedral element. This eliminates the need for boundary conforming meshes and opens the door for a seamless integration of patient-specific CT data into finite element analysis. In addition, voxel FEM guarantees the resolution of the complete microstructure available in the CT scans. At the same time, its linear basis functions do not over-resolve sharp interfaces and re-entrant corners between single voxels, which are an artifact of the geometric model. In combination with appropriate constitutive laws, e.g., based on plasticity [29], voxel FEM has been shown to accurately predict the evolution of bone failure [30,31]. Based on a large number of independent validation studies, it has established itself as a gold standard in the biomechanics community, where it is widely considered fit for use in clinical practice. Voxel FEM, however, involves a prohibitive computational cost when applied to CT scans of a complete bone.

The voxel finite cell method (FCM) [32–36] approximates the solution fields on a simple mesh that does not have to conform to the geometric boundaries of the object to be analyzed. Instead, the geometry is captured implicitly by means of special voxel-based quadrature rules. The voxel FCM finds the location of each quadrature point in the voxel model, eliminating the stiffness at quadrature points where no bone material exists. It thus implicitly accounts for the bone geometry during integration of the stiffness matrix. The voxel FCM has been successfully applied for patient-specific bone simulations in the linear elastic range [32–34,37,38], including validation with in-vitro experiments and high-order FEM [39–41].

In this paper, we develop a multiscale discretization approach that combines the advantages of the voxel FEM and the voxel FCM. Its goal is to significantly reduce the computational cost with respect to full-resolution voxel FEM, while maintaining the same accuracy level as well as the seamless integration of voxel data. In our approach, we use the finite cell method on a coarse higher-order mesh for representing elastic behavior that dominates the largest part of the structure. In areas, where we expect failure to occur, we add a fine-scale voxel mesh of linear hexahedral elements to accurately capture microstructure effects up to the available image resolution. In each coarse-scale cell, the fine-scale voxel mesh is adaptively switched on, as soon as a material model related yield indicator anticipates the possibility of inelastic deformations within the corresponding cell domain. From a computational point of view, we benefit from the increased efficiency of the voxel FCM. At the same time, we still use voxel FEM in regions with inelastic deformations, fully leveraging the substantial validation efforts and the wide acceptance of voxel FEM in the biomechanics community [42–44]. We perceive each coarse-scale cell associated with a fine-scale voxel mesh as two different scales that can be connected via scale interaction mechanisms, for which we define specific displacement and stress projectors [45–50].

We choose the displacement projector in such way that it automatically eliminates the communication between fine-scale voxel elements across the boundaries of coarse-scale cells. This enables an efficient predictor/corrector scheme that replaces the fully coupled system by a series of small uncoupled systems, which can be solved iteratively and in parallel with a fraction of computing power and memory. In addition, we show that our multiscale approach yields the same accuracy as the full-resolution voxel FEM, if we appropriately balance the approximation power of coarse-scale and fine-scale meshes. We emphasize that in this work, we do not focus on finding the most appropriate material model for bone failure, but provide a multiscale discretization framework, in which a wide variety of commonly used material models based on plasticity or damage mechanics [29] can be easily integrated. In this paper, without loss of generality, we use a von Mises plasticity model, which is widely accepted for modeling bone failure [3,51,52].

Our paper is organized as follows: in Section 2, we review relevant basic concepts of the voxel FCM and the voxel FEM. In Section 3, we establish the multiscale predictor/corrector scheme and its mechanisms that enable significant gains in computational efficiency. In Section 4, we present numerical examples that illustrate the accuracy of our approach, with a specific focus on bounding the mesh size ratio between coarse- and voxel-scale meshes and predicting the strength of a patient-specific vertebra. We draw conclusions in Section 5.