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A component-based hybrid reduced basis/finite element method for solid mechanics with local nonlinearities☆

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Abstract

The SCRBE (Static-Condensation Reduced-Basis-Element) method is a component-to-system model order reduction approach for efficient many-query and real-time treatment of linear partial differential equations characterized by many spatially distributed constitutive, geometry, and topology parameters. In this paper we incorporate the SCRBE approach into a framework for analysis of problems in solid mechanics which are largely linear with the exception of local nonlinearities. In particular, we exploit a linear–nonlinear domain decomposition to develop a hybrid formulation: we consider a SCRBE approximation over the (assumed predominant) part of the domain associated to a linear elasticity model; we revert to a full finite element (FE) approximation over the part of the domain associated to the locally nonlinear model. We adapt the SCRBE port training procedures to anticipate the behavior of the field over the linear–nonlinear interface and hence ensure an accurate solution over the entire domain. We choose a globally conforming approximation which exploits the intrinsic port structure of the SCRBE method and yields a decoupled "non-invasive" formulation of the respective linear and nonlinear blocks of the residual vector and (Newton) Jacobian matrix. We present numerical results for several local nonlinearities – elastic contact with and without friction, plastic (contact) – which demonstrate substantial savings relative to standard FE approximation over the entire domain, in particular in the regime in which the linear part of the domain represents the majority of the (FE) degrees of freedom.

Keywords: Partial differential equations; Reduced basis method; Port training; Components; Finite element method; Contact analysis; Plasticity

1. Introduction

Parametric model order reduction is an effective approach for the treatment of parametrized partial differential equations (PDEs) in the real-time and many-query contexts. Several approaches to parametrized model order reduction

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are in widespread use, including reduced basis (RB) methods [1–4], the Proper Orthogonal Decomposition (POD) [5], and the Proper Generalized Decomposition (PGD) [6]. (We restrict attention here to time-independent problems; for a more complete review of parametric model order reduction, with emphasis on dynamical systems, we refer to [7].) In this paper we focus on RB methods due to the stronger norm in parameter, and also efficiency, associated with the Weak Greedy sampling procedure [8,9], however there are certainly contexts in which the POD or PGD approaches are preferred.

As typically practiced, the "single-domain" RB method suffers from several limitations which preclude application to industrial-scale engineering problems:

1. A single-domain RB method only admits parametrizations which induce continuous dependence of the PDE solution on parameter. Discontinuous parametrizations – for example to describe topology variations, often crucial in engineering analysis and design – cannot be treated.
2. The single-domain RB method requires, at each parameter value proposed by the Weak Greedy procedure, solution of a costly underlying finite element (FE) approximation. In many industrial contexts, for example large structures which perforce exhibit a wide range of scales, even a single FE solution may be prohibitively expensive.
3. The single-domain RB method is restricted to relatively few parameters. For problems with many parameters, both the offline cost associated with an extensive training set, and the online cost associated with a rich reduced basis space, largely eliminate any benefit of model order reduction.

We refer to the above as single-domain RB shortcomings.

We next recall the SCRBE (Static-Condensation Reduced-Basis-Element) method [10–13]. The SCRBE approach comprises three principal ingredients: component-to-system synthesis, formulated as a Static-Condensation (SC) [14] procedure; model order reduction, informed by evanescence arguments at component interfaces (port reduction by spectral truncation) and low-dimensional parametric manifolds in component interiors (bubble reduction by RB approximation); and offline–online computational decomposition strategies based on affine or approximate affine expansions in parameter [15]. In practice, both the offline and online stages benefit from parallel implementation (parallel computation can also effectively serve in the single-domain RB context [16]). We note that the SCRBE model reduction, just as single-domain RB model reduction, is effected relative to – built upon – an underlying FE approximation.

The SCRBE approach addresses the three single-domain RB shortcomings: (online) changes in topology are readily accommodated through component instantiation and connection; both bubble and port training procedures are typically performed on one-component and two-component systems, and hence we avoid large FE calculations even in the offline stage; many applications are characterized by spatially distributed parameters, and hence the number of parameters associated with any given component is relatively small—divide and conquer.

The SCRBE method may be viewed as a combination of the Component Mode Synthesis (CMS) method [17–19] and the reduced basis method [1–4]: the CMS method provides the foundation of components, systems, and port and bubble reduction; the RB method provides the framework for efficient parametric analysis. A synthesis of CMS and RB approaches is first introduced in the Reduced Basis Element method (RBE) of [20]. The SCRBE method may thus be interpreted as an RBE method for the particular choice of Static Condensation (SC) interface treatment [10] and particular strategies for port training [11–13] and bubble training [8,9]. (For other applications of domain decomposition to model order reduction we refer to [21–25].)

The SCRBE method, and in particular the Static Condensation elimination of component-interior degrees of freedom, is intrinsically limited to linear problems. However, we can nevertheless *incorporate* the SCRBE method into nonlinear analyses. In this paper we shall consider a class of problems in which the spatial domain Ω can be decomposed as $\overline{\Omega} = \overline{\Omega}_{\mathrm{LIN}} \cup \overline{\Omega}_{\mathrm{NLIN}}$ such that the field is acted upon by a linear operator over Ω_{LIN} – in our case, linear elasticity – and a nonlinear operator over Ω_{NLIN} – in our case contact or plasticity. The linear–nonlinear domain decomposition now admits a hybrid formulation: we consider a "linear" SCRBE approximation over Ω_{LIN}; we consider a "nonlinear" approximation (more precisely, an approximation suitable for nonlinear operators) over Ω_{NLIN}. We shall be interested in local nonlinearities, for which the linear region Ω_{LIN} is as large as, and preferably substantially larger than, the nonlinear region Ω_{NLIN}; "linear predominance" is often realistic for contact and plasticity, in particular for structures in service (though not necessarily for forming processes).