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A discontinuous Galerkin method with Lagrange multipliers for spatially-dependent advection-diffusion problems

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Abstract

A higher-accuracy discontinuous Galerkin method with Lagrange multipliers (DGLM) is presented for the solution of the advection-diffusion equation with a spatially varying advection field in the high Péclet number regime, where the classical polynomial finite element method (FEM) produces spurious oscillations in the solution at practical mesh resolutions. The proposed DGLM method is based on discontinuous polynomial shape functions that are attached to an element rather than its nodes. It overcomes the aforementioned spurious oscillation issue by enriching these functions with approximate free-space solutions of homogeneous equations derived from an asymptotic analysis of the governing partial differential equation inspired by Prandtl's boundary layer theory. These enrichment functions are capable of resolving exponential, parabolic, and corner boundary layers at relatively coarse mesh resolutions. The proposed method enforces a weak continuity of the solution approximation across inter-element boundaries using polynomial Lagrange multipliers, which makes it a hybrid method. However, unlike other hybrid methods, it operates directly on the second-order form of the advection-diffusion equation and does not require any stabilization. Its intrinsic performance and its superiority over the higher-order polynomial FEM are demonstrated for several test problems at Péclet numbers ranging from one thousand to one billion.

Keywords: Advection-diffusion; Discontinuous Galerkin method; Enriched finite element method; Hybrid method; Lagrange multipliers

1. Introduction

Advection-diffusion problems are often used as model problems for the (incompressible) Navier-Stokes equations, particularly during the early development of discretization methods for these equations. They are typically characterized by the dimensionless Péclet number Pe, which is defined as the ratio of the advection and diffusion rates. This number is small ($Pe \ll 1$) for diffusion-dominated flows, and large ($Pe \gg 1$) for advection-dominated ones. In the diffusion-dominated flow regime, the standard polynomial finite element method (FEM) delivers a

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