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PII: S0045-7825(17)30648-5
DOI: https://doi.org/10.1016/j.cma.2017.09.018
Reference: CMA 11612

To appear in: *Comput. Methods Appl. Mech. Engrg.*

A New Goal-Oriented Formulation of the Finite Element Method

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September 23, 2017

Abstract

In this paper, we introduce, analyze, and numerically illustrate a method for taking into account quantities of interest during the finite element treatment of a boundary-value problem. The objective is to derive a method whose computational cost is of the same order as that of the classical approach for goal-oriented adaptivity, which involves the solution of the primal problem and of an adjoint problem used to weigh the residual and provide indicators for mesh refinement. In the current approach, we first solve the adjoint problem, then use the adjoint information as a minimization constraint for the primal problem. As a result, the constrained finite element solution is enhanced with respect to the quantities of interest, while maintaining near-optimality in energy norm. We describe the formulation in the case of a problem defined by a symmetric continuous coercive bilinear form and demonstrate the efficiency of the new approach on several numerical examples.

Keywords: Finite Element Method, Goal-Oriented Formulation, Mixed Formulation, Error Estimation, Adaptive Mesh Refinement, Multi-Objective Goal Functionals

1 Introduction

Advances in Computational Science and Engineering have reached such a level of maturity that increasingly complex multiphysics and multiscale problems can now be simulated for decision-making and optimal design. The focus of such simulations has thus shifted towards efficiently and accurately predicting specific features of the solution rather than the whole solution itself. With that objective in mind, goal-oriented error estimation and adaptive methods [20, 22], whose predominant instance is the dual-weighted residual method [5], have been developed since the late nineties in order to estimate and control errors with respect to quantities of interest. The principle of these methods essentially relies on the solution of adjoint problems associated with quantities of interest in order to identify and refine the sources of discretization or modeling errors that influence