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A new stochastic simulation algorithm for updating robust reliability of linear structural dynamic systems subjected to future Gaussian excitations

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Highlights

- Literature establishes motives behind interest in Robust reliability updating.
- Structural modeling and stochastic excitation modeling uncertainties are considered.
- Approach is robust to the number of random variables and the dimension of modal data.
- New algorithm is proposed to simulate samples from conditional distribution.

Abstract

In this paper, we are interested in using system response data to update the robust failure probability that any particular response of a linear structural dynamic system exceeds a specified threshold during the time when the system is subjected to future Gaussian dynamic excitations. Computation of the robust reliability takes into account uncertainties from structural modeling in addition to the modeling of the uncertain excitations that the structure will experience during its lifetime. In partial, modal data from the structure are used as the data for the updating. By exploiting the properties of linear dynamics, a new approach based on stochastic simulation methods is proposed to update the robust reliability of the structure. The proposed approach integrates the Gibbs sampler for Bayesian model updating and Subset Simulation for failure probability computation. A new efficient approach for conditional sampling called 'Constrained Metropolis within Gibbs sampling' algorithm is developed by the authors. It is robust to the number of uncertain parameters and random variables and the dimension of modal data involved in the problem. The effectiveness and efficiency of the proposed approach are illustrated by two numerical examples involving linear elastic dynamic systems.

Keywords: Reliability updating; Stochastic simulation; Uncertainty quantification; Structural reliability

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1. Introduction

The condition of a structure may change from time to time during its operation and may deteriorate which may lead to a significant reduction of its reliability. Therefore, it is essential to assess the reliability of a dynamic system from time to time during its operation. In real practice, it is impossible to measure the physical properties of the system directly in space and time. The estimates of the model parameters of the mathematical model used to represent the behavior of the real structure always involve uncertainties due to the limitations of the model in representing reality and the presence of measurement error in the data etc. Reliability considering model uncertainties in addition to the modeling of the uncertain excitations is termed as robust reliability [1].

Let $\boldsymbol{\theta}_s \in \mathbb{R}^n$ denote the uncertain model parameter vector specified by a model class $\mathcal{M}$ with the initial/prior probability density function (PDF) $p(\boldsymbol{\theta}_s|\mathcal{M})$ and future dynamic input specified by a stochastic input model $\mathcal{U}$ which can be expressed as a linear combination of a finite number of independently and identically distributed (i.i.d) standard normal random variables $\mathbf{Z} \in \mathbb{R}^{N_T}$. The robust reliability or its complement the robust failure probability is given by the following multi-dimensional integral with respect to $\boldsymbol{\theta}_s$ and $\mathbf{Z}$:

$$P(F|\mathcal{M}, \mathcal{U}) = \int I_F(\boldsymbol{\theta}_s, \mathbf{Z}) p(\boldsymbol{\theta}_s|\mathcal{M}) p(\mathbf{Z}|\mathcal{U}) d\mathbf{Z} d\boldsymbol{\theta}_s \tag{1}$$

where F denotes the limit state: $F \equiv \{D(\boldsymbol{\theta}_s, \mathbf{Z}) > b\}$ defines ‘failure’ with $D(\boldsymbol{\theta}_s, \mathbf{Z})$ being the performance function and b the corresponding failure threshold. $I_F \equiv I_F(\boldsymbol{\theta}_s, \mathbf{Z})$ is the indicator function, such that $I_F = 1$ if $D(\boldsymbol{\theta}_s, \mathbf{Z}) > b$ and $I_F = 0$ otherwise.

Most vibration data of structural systems under investigation are obtained under low-amplitude excitation (such as ambient vibration data). Thus, it can be assumed that during the time when the vibration data are collected, the structural system, even an already damaged one, behaves approximately linearly. Given the measurement data D from the system, the updated/posterior distribution of $\boldsymbol{\theta}_s$ is given by Bayes’ theorem as follows:

$$p(\boldsymbol{\theta}_s|D, \mathcal{M}) = \frac{p(D|\boldsymbol{\theta}_s, \mathcal{M}) p(\boldsymbol{\theta}_s|\mathcal{M})}{p(D|\mathcal{M})} \tag{2}$$

where $p(D|\boldsymbol{\theta}_s, \mathcal{M})$ is the likelihood function based on the predictive PDF of the response given by model class $\mathcal{M}$, and $p(D|\mathcal{M})$ is the normalizing constant which makes the probability volume under the posterior distribution equal to unity.

The updated robust failure probability given D is then given by replacing the prior distribution $p(\boldsymbol{\theta}_s|\mathcal{M})$ in (1) with the posterior distribution $p(\boldsymbol{\theta}_s|D, \mathcal{M})$ in (2):

$$P(F|D, \mathcal{M}, \mathcal{U}) = \frac{\int I_F(\boldsymbol{\theta}_s, \mathbf{Z}) p(D|\boldsymbol{\theta}_s, \mathcal{M}) p(\boldsymbol{\theta}_s|\mathcal{M}) p(\mathbf{Z}|\mathcal{U}) d\mathbf{Z} d\boldsymbol{\theta}_s}{\int p(D|\boldsymbol{\theta}_s, \mathcal{M}) p(\boldsymbol{\theta}_s|\mathcal{M}) d\boldsymbol{\theta}_s}. \tag{3}$$

There are several difficulties in evaluating the above integral. It can be expected that the dimension of the above integral is high due to a large number of random variables involved and the failure region in $\boldsymbol{\theta}_s$ and $\mathbf{Z}$ space has complicated geometry, and thus it will be impossible to analytically evaluate the integral. For $P(F|D, \mathcal{M}, \mathcal{U}) << 1$ and high-dimensional data D, it is often infeasible to evaluate the integrals in the numerator (and denominator) of (3) by simulation based methods such as Monte Carlo Simulation (MCS) or importance sampling since the high-probability content region of their corresponding integrands may occupy a much smaller volume than that of the prior joint distribution for $\boldsymbol{\theta}_s$ and $\mathbf{Z}$, i.e., $p(\boldsymbol{\theta}_s|\mathcal{M}) p(\mathbf{Z}|\mathcal{U})$ (and that of the prior distribution of $\boldsymbol{\theta}_s$, i.e., $p(\boldsymbol{\theta}_s|\mathcal{M})$).

Over the past few years, several methods have been presented to tackle the aforementioned difficulties in evaluating the robust failure probability. Papadimitriou et al. [1] presented an approach based on Laplace’s asymptotic approximation to compute the robust failure probability. However, this approach can be computationally challenging in a high-dimensional parameter space and can be inaccurate when the Gaussian assumption for the posterior PDF is not valid for the global identifiable case. Beck and Au [2] proposed a level-adaptive Metropolis–Hastings algorithm with a global proposal PDF to obtain the samples from the posterior PDF and then use these samples to update the system reliability by evaluating the system reliability conditional on each of these samples. The approach will experience difficulty when the number of uncertain model parameters is large and is computationally inefficient because it requires multiple reliability analyses equal to the number of posterior samples. Ching and Beck [3] proposed a method to update the reliability based on combining a Kalman filter and smoother, and modifying the algorithm ISEE [4]. Such an approach is only applicable to linear dynamic systems with no uncertainties in model parameters. Ching and Hsieh [5] proposed a method based on Bayes’ theorem and an analytical approximation of some of the