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On the fixed-stress split scheme as smoother in multigrid methods for coupling flow and geomechanics

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Abstract

The fixed-stress split method has been widely used as solution method in the coupling of flow and geomechanics. In this work, we analyze the behavior of an inexact version of this algorithm as smoother within a geometric multigrid method, in order to obtain an efficient monolithic solver for Biot's problem. This solver combines the advantages of being a fully coupled method, with the benefit of decoupling the flow and the mechanics part in the smoothing algorithm. Moreover, the fixed-stress split smoother is based on the physics of the problem, and therefore all parameters involved in the relaxation are based on the physical properties of the medium and are given a priori. A local Fourier analysis is applied to study the convergence of the multigrid method and to support the good convergence results obtained. The proposed geometric multigrid algorithm is used to solve several tests in semi-structured triangular grids, in order to show the good behavior of the method and its practical utility.

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Keywords: Iterative fixed-stress split scheme; Smoother; Multigrid; Poroelasticity; Local Fourier analysis

1. Introduction

In recent years, intensive research has been focused on the design of efficient methods for solving the large linear systems arising from the discretization of Biot's model, since in real simulations it is the most consuming part. Using an implicit time-stepping discretization, the resulting system matrix on each time step is an example of saddle point problem requiring therefore specific solvers. There are mainly two approaches, the so-called monolithic or fully coupled methods and the iterative coupling methods. In the monolithic approach, the linear system is solved simultaneously for all the unknowns, and it usually provides unconditional stability and convergence. The challenge here is the design of efficient preconditioners to accelerate the convergence of Krylov subspace methods and the

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design of efficient smoothers in a multigrid framework. Recent advances in both directions can be found in [1–4] and the references therein. On the other hand, at each time step, iterative coupling methods solve sequentially the equations for fluid flow and geomechanics, until a converged solution within a prescribed tolerance is achieved. A big advantage of these methods is their flexibility since two different codes for fluid flow and geomechanics can be linked for solving the poroelastic problems. The most used iterative coupling methods are the drained and undrained splits, which solve the mechanical problem first, and the fixed-strain and fixed-stress splits, which on the contrary solve the flow problem first [5,6].

Due to its unconditional stability, one of the most frequently used schemes of this type is the so-called fixed-stress split method. This sequential-implicit method basically solves the flow problem first by fixing the volumetric mean total stress, and then the mechanics part is solved from the values obtained at the previous flow step. The unconditional stability of the fixed-stress split method is shown in [7] using a von Neumann analysis. In addition, stability and convergence of the fixed-stress split method have been rigorously established in [8]. Recently, in [9] the authors have proven the convergence of the fixed-stress split method in energy norm for heterogeneous problems. Estimates for the case of the multirate iterative coupling scheme are obtained in [10], where multiple finer time steps for flow are taken within one coarse mechanics time step, exploiting the different time scales for the mechanics and flow problems. In [11], the convergence of this method is proven in the fully discrete case when space–time finite element methods are used. In [12], the authors present a very interesting approach which re-interprets the fixed-stress split scheme as a preconditioned-Richardson iteration with a particular block-triangular preconditioning operator. In fact, the four commonly used sequential splitting methods, i.e. drained-split, undrained-split, fixed-stress and fixed-strain [5], can be seen in this way [13]. It is analyzed that a fully-implicit method outperforms the convergence rate of the sequential-implicit methods. Following this approach a family of preconditioners to accelerate the convergence of Krylov subspace methods has been recently proposed for the three-field formulation of the poromechanics problem [14].

Here, we want to propose the use of an inexact version of the fixed-stress split scheme as a smoother in a geometric multigrid framework, in order to obtain an efficient monolithic solver for Biot's problem. This approach combines the advantages of being a fully-coupled method on the one hand with the benefit of decoupling the flow and the mechanics part in the smoothing algorithm on the other hand. Recently, another decoupled smoother, based on an inexact Uzawa method, has been successfully proposed to solve the poroelastic problem in a monolithic manner [4]. However, the key for a satisfactory performance of this smoother is the choice of a relaxation parameter in the pressure update step. The main advantage of the fixed-stress split smoother proposed here is that the parameter is established by the physics of the problem, opposite to the case of the Uzawa smoother for which the relaxation parameter has to be carefully chosen.

The remainder of the paper is organized as follows. Section 2 is devoted to the description of the poroelasticity model and the considered finite element discretization. In Section 3, after a brief introduction of the fixed-stress split algorithm, we introduce the class of smoothers that we proposed based on that method, and we analyze their relaxation properties by using a smoothing local Fourier analysis technique. In Section 4, the rest of the multigrid components that we consider are listed and a two-grid local Fourier analysis is used to study the convergence of the resulting multigrid algorithm. Also in this section, the implementation of the geometric multigrid on semi-structured grids is explained in order to extend the proposed method to problems on more complex domains. Section 5 illustrates the good convergence of the multigrid method based on the fixed-stress split smoothers through two numerical experiments, one of them considering also variable coefficients. Finally, some conclusions are drawn in Section 6.

2. Mathematical model and discretization

We begin with a small introduction about the quasi-static Biot's model for soil consolidation. For a more detailed explanation about the governing equations and the mathematical model we refer to the reader to the books by Wang [15] and Coussy [16], for instance. A stabilized P1–P1 finite element method is considered to describe the solver proposed in this work as well as to present the numerical experiments. However, our approach can be easily implemented for other finite-element and finite-volume discretizations.