Available online at www.sciencedirect.com
ScienceDirect
Comput. Methods Appl. Mech. Engrg. 326 (2017) 642–655
Computer methods in applied mechanics and engineering
www.elsevier.com/locate/cma

A direct simulation algorithm for a class of beta random fields in modelling material properties

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Received 20 April 2017; received in revised form 7 July 2017; accepted 4 August 2017
Available online 26 August 2017

Highlights

- Non-translation algorithm for a class of beta random fields.
- Autocorrelation function is invariant with respect to the marginal beta distribution.
- Simple and efficient approach in generating large-scale 3D random fields.

Abstract

Numerous translation approaches have been developed to generate non-Gaussian random fields; however, non-translation or direct simulation methods are much rare. The translation approaches invoke the memoryless transformation from an underlying Gaussian random field to a target non-Gaussian random field. The correlation structure is changed by the memoryless transformation from a Gaussian to a non-Gaussian field and as a result the non-Gaussian correlation structure is obtained after amending the Gaussian correlation structure. In addition, the underlying Gaussian random field is often generated by series expansion approaches (e.g. spectral representation method), which require a sum of infinite terms. Consequently, the computational efforts involved in generating the Gaussian random fields can be significant. After considering the difficulties in simulating a translation random field, a direct approach without utilizing the memoryless transformation is proposed to generate a beta random field. Though the marginal distribution is restricted to the beta distribution, the proposed approach is simple and efficient. This would make it attractive in simulating large-scale random fields for material properties. This is exemplified with an engineering application.

Keywords: Material property; Spatial variability; Random field; Beta distribution; Autocorrelation function; Monte Carlo simulation

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http://dx.doi.org/10.1016/j.cma.2017.08.001

1. Introduction

The presence of uncertainties in material properties has been acknowledged by the engineering community and accounted for in codes of practice [1,2]. By treating the material properties as random variables or in some cases as random fields, statistics-based analysis can be found in a consistent manner [3–6]. The four-parameter beta distribution is widely used to describe the variation in material properties (e.g. [7,8]). In comparison to other commonly used distributions, such as the normal and lognormal distributions; the beta distribution is advantageous in its limited bounds and flexible shape in data fitting. By controlling the mean, variance, lower and upper bounds, the skewness and kurtosis can also be reflected with the beta distribution [9]. Furthermore, the beta distribution has well-established properties for computer-aided simulations. For example, the MatLab software can generate beta random numbers. These features make the beta distribution a better alternative to other distributions which can also deal with high order moments, such as the maximum entropy distribution [10], Hermite polynomials of normal variates [11] and Johnson's system [12]. The high order statistics (e.g. skewness and kurtosis) usually require a large volume of data to estimate; by contrast, the limit in bounds could be more readily estimated by physical judgment. A typical example is the friction angle of a Mohr–Coulomb material. The range of a friction angle should be physically restricted from zero to $\pi/2$. Harrop-Williams has analytically revealed that the tangent of friction angle of soils follows the beta distribution [13].

The material properties are usually auto-correlated; whereby, two neighbouring points tend to have similar properties. In this regard, the concept of random fields is often utilized, which is controlled by a marginal probability distribution and an autocorrelation function (ACF). Currently, the techniques available for generating random fields with a marginal beta distribution are mainly based on the translation theory [11]. This invokes the memoryless nonlinear transformation of an underlying Gaussian random field. A stationary beta random field can be obtained based on the memoryless nonlinear translation provided that the underlying Gaussian field is stationary. The requirement of Gaussianity and stationarity of the underlying Gaussian field make the computational efficiency relatively low. Common techniques for generating the underlying Gaussian field are the spectral representation method (SRM [14]) and the Karhunen–Loève (KL) expansion [15,16]. Both methods involve a sum of infinite terms, which is unachievable in practice. With finite terms (say m terms in a one-dimensional problem), either the stationarity or the Gaussianity cannot be guaranteed. In terms of material properties, 2-D and 3-D models are often used, which as a result exponentially increases the number of sum terms (i.e. m^2 and m^3) in generating a Gaussian random number. In order to make the computation tractable, m is unlikely to be arbitrarily large. As a result, one has to balance the computation effort and accuracy in simulation.

The correlation structure is another difficulty in simulating a translation random field. The memoryless transformation changes the correlation structure from a Gaussian to a non-Gaussian field. This results in the target non-Gaussian correlation structure to be only obtained by altering the underlying Gaussian correlation structure. Various approaches have been developed for the adjustment [17]; however, most of them are numerical and iterative methods which are used to approximately obtain a target correlation structure. A few attempts can be found in order to circumvent the changes in correlation structure by adopting non-translation methods (e.g. [15,18]). Phoon et al. [15] demonstrated that the KL expansion technique can be adjusted to generate a non-Gaussian random field with a target correlation structure, at the absence of an underlying Gaussian field. However, the computational efficiency problem is still applicable for the non-translation approach using the KL expansion; the latter involves a sum of infinite terms.

Taking these difficulties into consideration in simulating a translation random field, a direct approach is proposed to generate a beta random field without utilizing the memoryless transformation. This approach consists of a mathematical algorithm of Monte Carlo simulation. The ACF is aimed to be invariant with the marginal distribution, so that the ACF will not change when the mean and variance change. Although the marginal distribution is restricted to the beta distribution, the proposed approach is simple and efficient, which would therefore make it attractive in simulating large-scale random fields for material properties.

The paper is organized as follows. Section 2.1 introduces three beta random variables that will be utilized in generating a beta random field. Further on, Section 2.2 proposes a Monte Carlo simulation algorithm of a beta random process, and the probabilistic characteristics of the proposed algorithm are discussed in detail. The algorithm is then extended to a general form in n–D spaces in Section 2.3. Discussions on the proposed algorithm are given in Section 3. Section 4 illustrates an engineering application where the proposed algorithm was used to stimulate the spatial variability in the strength of cement-treated soils in a 3-D column. This is where finite element analysis and Monte Carlo simulations together are used to evaluate the strength of the full-scale column in a statistical manner.