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Condensed SFEs for nonlinear mechanical problems

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Abstract

This paper introduces a coupled approach between stochastic finite element methods and an adaptive condensation technique for the analysis of nonlinear mechanical problems under uncertainties. This coupling reduces the size of each individual nonlinear problem solved in SFE by the use of an adaptive condensation method. The reduced stiffnesses and other quantities necessary for the condensation technique are approximated using a second, low-order, polynomial expansion, thus taking advantage of the coupling with SFE. This approach also features a semi-analytical technique to compute accurately distributions of structural quantities of interest. This method is applied on an elasto-plastic steel bar with a small defect, and on a damaged beam under 4-point bending. In both cases it predicts the random behavior of the structure quite accurately, and is able to provide higher-order models than a state-of-the-art stochastic collocation method, for a reduced computation time.

Keywords: Stochastic finite elements; Concrete cracking; Collocation; Static condensation; Metamodels

1. Introduction

During the last decades, numerous methods have been developed, inside the general framework of stochastic finite elements (SFE). These methods allow the use of finite element analysis on uncertain mechanical problems. They can also quantify the influence on the structure of these uncertainties on quantities of interest. The Monte-Carlo (MC) method [1,2] should be considered as the first one, and is still widely used in different application domains. However, more recent modeling techniques offer, compared to the MC method, higher orders of convergence, and require a reduced number of deterministic simulations [3–6]. These techniques are now used in several industrial simulation codes [7,8]. In certain cases, they even provide directly approximations of the statistical quantities of interest [9–11]: mean values, statistical moments, probability density functions (PDFs), etc. SFE methods are characterized by three combined building blocks, which allow to predict the behavior of a structure under uncertainties [6].

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- A model for uncertainties (e.g. modeling of random material properties).
- A mathematical technique to solve deterministic problems (typically, direct FE simulation).
- An algorithm to propagate deterministic solutions depending on the uncertainties (e.g. perturbation, polynomial chaos expansion).

Various methods have been developed to model the uncertainties on geometry, material properties, loading. Techniques to propagate uncertainties through deterministic solutions have also been widely studied, and various classes of methods are now available in literature. However, in the framework of SFE, few studies addressed the problem of the model and of the numerical techniques to solve the deterministic problems, in particular to minimize the computational cost [6,12]. It is especially the case for large scale nonlinear systems, for which non intrusive approaches such as Polynomial-Chaos or collocation-based methods are generally preferred to intrusive ones.

This work presents a coupled method to solve nonlinear SFE problems at a reduced cost. It includes a deterministic system reduction approach similar to those in [13] (using dynamic condensation) or in [14] (using static condensation). However, the presented method:

- uses an adaptive condensation technique based on a two-level Guyan's reduction, suitable for nonlinear mechanical problems [15,16].
- builds a second, lower-order, metamodel. This is used for the stiffnesses, equivalent loadings and displacements fields necessary in the condensation technique.
- provides full probability density functions (PDF) of quantities of interest thanks to a semi-analytical approach or Monte-Carlo simulations when it is not possible.

This method may be used in mechanical engineering, to evaluate the robustness of numerical models, build fragility curves of mechanical systems, etc. In particular, an application to civil engineering structures is proposed in this contribution.

Firstly, the nonlinear stochastic mechanical problem to be solved is presented and formulated. Then, the proposed method to solve it efficiently is described. A validation is performed on an heuristic test case, where the presented SFEM is compared to analytical results. Finally, an application is presented, where the method is compared to a state-of-the-art SFE technique.

2. Problem setting

2.1. Stochastic nonlinear mechanical problem

The problem is a general n-dimensional nonlinear mechanical problem, defined on a bounded domain $\Omega \in \mathbb{R}^d$ ($d = 1, 2, 3$). It is characterized by a set of r input parameters $y = (y_1, \ldots, y_r)$. It is assumed that the problem is well posed. The problem is solved using the finite element method (implemented in Cast3M [17]). The resulting n-dimensional mechanical system is considered through a pseudo-time t. Using a discretized time τ, the problem can be solved iteratively:

$$\left[K^{\tau}(y, u^{\tau}(y))\right] \cdot \left[u^{\tau}(y)\right] = \left[F^{\tau}(y)\right] \tag{1}$$

where $K^{\tau} \in \mathbb{R}^{n,n}$ is the stiffness matrix, $u^{\tau} \in \mathbb{R}^n$ the nodal displacement vector, and $F^{\tau} \in \mathbb{R}^n$ the nodal force vector (at time $t = \tau$). Mechanical quantities of interest can be extracted once the structural problem has been solved: they are considered as a random vector $Z = h^{\tau}(y)$. We consider in the following that the random input parameters can be normalized to a standardized Gaussian random vector $x = (x_1, \ldots, x_r) = T(y)$, using the Gaussian standardization function T [18]. Given that any distribution can be generated from a standard distribution [18], this methodology allows to chose any type of law for the actual r.v.s used in the problem (mechanical properties, random fields, etc.) with a single SFE method. The denormalization function is therefore included in the mechanical response function $\mathcal{M}$. The random vector Z writes:

$$Z = h^{\tau}(y) = h^{\tau} \circ T^{-1}(x) = \mathcal{M}(x). \tag{2}$$

2.2. Construction of a metamodel

The input uncertain parameters y are represented using a r-dimensional vector of standard independent random variables (r.v.s) $X = T(Y) = (X_1, \ldots, X_r)$, defined on the probability space $(\Theta, \mathcal{F}, \mathcal{P})$.