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A diffuse interface model for two-phase ferrofluid flows[☆]

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Abstract

We develop a model describing the behavior of two-phase ferrofluid flows using phase field-techniques and present an energy-stable numerical scheme for it. For a simplified, yet physically realistic, version of this model and the corresponding numerical scheme we prove, in addition to stability, convergence and as by-product existence of solutions. With a series of numerical experiments we illustrate the potential of these simple models and their ability to capture basic phenomenological features of ferrofluids such as the Rosensweig instability.

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1. Introduction

A ferrofluid is a liquid which becomes strongly magnetized in the presence of applied magnetic fields. It is a colloid made of nanoscale monodomain ferromagnetic particles suspended in a carrier fluid (water, oil, or other organic solvent). These particles are suspended by Brownian motion and will not precipitate nor clump under normal conditions. Ferrofluids are dielectric (non conducting) and paramagnetic (they are attracted by magnetic fields, and do not retain magnetization in the absence of an applied field); see [1].

Ferrofluids can be controlled by means of external magnetic fields, which gives rise to a wealth of control-based applications. They were developed in the 1960's to pump fuel in spacecrafts without mechanical action [2]. Recent interest in ferrofluids is related to technical applications such as instrumentation, vacuum technology, lubrication, vibration damping, radar absorbing materials, and acoustics [3–5]; they are used, for instance, as liquid seals for the

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drive shafts of hard disks, for vibration control and damping in vehicles and enhanced heat transfer of electronics. Other potential applications are in micro/nanoelectromechanical systems: magnetic manipulation of microchannel flows, particle separation, nanomotors, micro electrical generators, and nanopumps [6–10]. One of the most promising applications is in the field of medicine, where targeted (magnetically guided) chemotherapy and radiotherapy, hyperthermia treatments, and magnetic resonance imaging contrast enhancement are very active areas or research [11–13]. Yet another potential applications of ferrofluids under current research is the construction of adaptive deformable mirrors [14–16].

At the time of this writing there are two well established PDE models as a mathematical description for the behavior of ferrofluids which we will call by the name of their developers: the Rosensweig model and the Shliomis model (cf. [17,18]). Rigorous mathematical work on the mathematical analysis (existence of global weak solutions and local existence of strong solutions) for the Rosensweig and the Shliomis models is very recent (cf. [19–22]).

The applications mentioned above justify the development of tools for the simulation of ferrofluids, but they are not the only reasons. Mathematical models for ferrofluids and their scope of validity have been areas of active research (cf. [23,24]). Most ferrofluid flows have so far been studied using exact and approximate analytical solutions of the Rosensweig model (see for instance [25]) contrasted with experimental data. However, these flows are analytically tractable in a very limited number of cases [25,26], and as shown for instance in [27], satisfactory model calibration/validation is beyond the current capabilities of analytic (asymptotic and perturbation) methods. Clearly, there is significant room for interdisciplinary work at the interface between model development, numerical analysis, simulation and experimentation.

Both the Rosensweig and Shliomis models deal with one-phase flows, which is the case of many technological applications. However, some applications arise naturally in the form of a two-phase flow: one of the phases has magnetic properties and the other one does not (e.g. magnetic manipulation of microchannel flows, microvalves, magnetically guided transport, etc.). There has been a major effort in order to develop appropriate interfacial conditions of two-phase flows in the sharp interface regime within the micropolar theory (see [28,29]), yet we are far from having at our disposal a mathematically and physically sound PDE model for two-phase ferrofluid flows. There are not well established PDE models describing the behavior of two-phase ferrofluid flows. On the other hand, systematic derivation of a two-phase model from first principles, using energy-variational techniques in the spirit of Onsager's principle as in [30–34], would be highly desirable, but most probably too premature, given the current state of the art.

In this context, numerical analysis and scientific computation have a lot to offer, since carefully crafted computational experiments can help understand much better the limits of the current models and assist the development of new ones. Ad-hoc development (trial and error) of new models and numerical evaluation does not replace a proper mathematical derivation, but it can clearly help to find a reasonable starting point. In this spirit, the main goal of this work is to present a simple two-phase PDE model for ferrofluids. The model is not derived, but rather assembled using components of already existing models and high-level (as opposite to deep) understanding of the physics of ferrofluids. The model attempts to retain only the essential features and mathematical difficulties that might appear in much more sophisticated models. To the best of our knowledge this contribution is the first modeling/numerical work in the direction of time-dependent behavior of two-phase ferrofluid flows together with energy-stable and/or convergent schemes.

Regarding pre-existing work, closely related to two-phase flows, it is worth mentioning the interdisciplinary (including physical experiments) work of Tobiska and collaborators [35–37] in the context of stationary configurations of free surfaces of ferrofluids using a sharp interface approach. Other models for two-phase ferrofluid flows, this time for non-stationary phenomena, are presented in [38–40], using either Level-Set or Volume of Fluid methods, but very little details are given about their actual numerical implementation, stability or convergence properties.

Our presentation is organized as follows: in Section 2 we select the components of our two-phase model and assemble it. In Section 3 we derive formal energy estimates which will serve as basis for the development of an energy-stable scheme in Section 4; in Section 4.3 we prove that the scheme always has a solution. In Section 5, we propose a simplification of the model with a more restrictive scope of physical validity. We present the corresponding numerical scheme, and prove its stability and convergence in Section 5.1 and Section 5.2 respectively. Finally, we show the potential of the model in Section 6 with a series of insightful numerical experiments. They include the Rosensweig instability with uniform and non-uniform applied magnetic fields, the latter leading to an open pattern of spikes.