Available online at www.sciencedirect.com

ScienceDirect

Comput. Methods Appl. Mech. Engrg. 308 (2016) 483–498

Computer methods in applied mechanics and engineering

www.elsevier.com/locate/cma

Research Paper

A general algorithm for evaluating nearly singular integrals in anisotropic three-dimensional boundary element analysis

Yan Gu[a,*], Hongwei Gao[a], Wen Chen[b], Chuanzeng Zhang[c]

[a] *College of Mathematics, Qingdao University, Qingdao 266071, PR China*

[b] *Department of Engineering Mechanics, College of Mechanics and Materials, Hohai University, Nanjing 210098, PR China*

[c] *Department of Civil Engineering, University of Siegen, Paul-Bonatz-Str. 9-11, D-57076 Siegen, Germany*

Received 20 January 2016; received in revised form 18 April 2016; accepted 27 May 2016
Available online 4 June 2016

Abstract

This paper presents a new general method for the evaluation of nearly singular boundary element integrals arising in anisotropic three-dimensional (3D) boundary element analysis. It is shown that the original nearly singular integrals can be transformed into, using a sinh function, an element-by-element sum of regular integrals, each one expressed in terms of intrinsic (local) coordinates. As a consequence, the actual computation can be performed by using standard $n \times n$ Gaussian quadrature and the procedure can be easily included in any existing computer code. This new method has full generality and, therefore, can be applied to a wide class of integrals. The numerical results demonstrate the accuracy and efficiency of the method, along with its insensitivity to the location of the nearly singular points. It is shown that several orders of magnitude improvement in relative errors can be obtained using this transformation when compared to a straightforward implementation of Gaussian quadrature.

Keywords: Boundary element method; Nearly singular integrals; Non-linear coordinate transformation; Three-dimensional anisotropic problems; Sinh function

1. Introduction

The finite element method (FEM) has long been a dominant numerical technique in the simulation of real-world engineering applications. However, this method requires the task of meshing the whole domain which can be arduous, time-consuming and computationally expensive for certain classes of problems [1]. As an alternative approach, the boundary element method (BEM) has long been touted to avoid such shortcomings due to the boundary-only discretizations and its semi-analytical nature. During the past two decades, the BEM has rapidly improved, and is nowadays considered as a competing method to the FEM. However, in order to achieve an efficient numerical implementation of general validity, a number of issues associated with the BEM have to be dealt with using special attention.

* Corresponding author. Fax: +86 25 8373 6860.
E-mail address: guyan1913@163.com (Y. Gu).

http://dx.doi.org/10.1016/j.cma.2016.05.032
0045-7825/

One of the typical and most significant issues of almost all BEM analyses is the accurate evaluation of nearly singular boundary element integrals [2–6]. These integrals are *nearly singular* in the sense that the calculation point is approaching towards, but not on, the boundary element. Theoretically, these integrals are regular since the values of the integrands are always finite. However, instead of remaining smooth, the integrand develops a sharp peak as the calculation point moves closer to the boundary. Accurate evaluation of such integrals faces considerable difficulties because neither the standard Gaussian quadrature nor the methods designed for singular integrals can be employed [7–9]. The accurate evaluation of nearly singular integrals plays an important role in many engineering applications, such as the study of thin structures [10,11], contact problems [12], sensitivity problems [13] and displacement around open crack tips [14].

Over the past two decades, some considerable effort was devoted to proposing novel computational algorithms that circumvent or greatly eliminate the near singularity issues associated with the boundary element analysis. Apart from pure analytical integration, which has obvious limitations (low order elements), many other methods have been devised. The methods developed so far include, but are not limited to, element subdivision methods [15–17], semi-analytical methods [6,18,19] and various nonlinear transformations [20–27]. The element subdivision method is appealing, stable, and accurate but is costly because the number of sub-elements and their sizes are strongly dependent on the order of the near singularity and the dimension of the element. The semi-analytical method is accurate but it is not available for a general distributed function and curved boundary surface because closed-form integrations of the boundary function are not possible [6]. The most popular techniques used currently to calculate the nearly singular integrals are various nonlinear transformations, for example, the rational transformation [21], the optimal transformation [24], the distance transformation [28], the sinh transformation [22,29–31], and the exponential transformation [32]. The common feature of these transformations is to remove or smooth out the near singularities of the integrand using a nonlinear function before the conventional Gaussian quadrature is applied.

Impressive results obtained from aforementioned techniques have been demonstrated on various examples. However, a number of drawbacks still remain and mainly include the fact that some techniques are restricted to nearly singular integrals defined on linear geometry elements while others are tailored to some certain kind of integrals. In addition, almost all the methods developed so far share the common feature of evaluating nearly singular integrals that only exist in isotropic BEM formulations. To date, very few studies on the calculation of nearly singular integrals arising in general anisotropic media have been reported in the BEM community [30].

Inspired by the pioneering work mentioned above, we present an extension of the previously published sinh transformation [30,33] for the evaluation of nearly singular integrals, which arise in general 3D anisotropic boundary element analysis. It will be demonstrated that the nearly singular integrals with anisotropic properties can be always transformed to regular ones through simple, although rigorous, manipulations. As a consequence, the actual computation can be performed by using standard $n \times n$ Gaussian quadrature. The new method is applicable to high-order (curved) geometry elements and can be applied whatever the type and order of the kernel function considered. It will be shown that the new method is easy to implement and could improve the accuracy of evaluating nearly singular integrals by several orders of magnitude, compared with a straightforward application of standard Gaussian quadrature.

A brief outline of the rest of the paper is as follows. The BEM formulation for general 3D anisotropic potential problems are described in Section 2. The nearly singular integrals and their numerical implementation are introduced in detail in Sections 3 and 4, respectively. Four benchmark examples that are commonly encountered in the applications of the BEM are examined in Section 5. Finally, the conclusions and remarks are provided in Section 6.

2. The BEM for potential problems in anisotropic bodies

Consider a 3D anisotropic medium in an open bounded domain Ω, and assume that Ω is bounded by a surface Γ which may consist of several segments, each being sufficiently smooth in the sense of Liapunov. In this study, we refer to anisotropic steady heat conduction applications in the absence of inner heat sources. Hence the function $u(\boldsymbol{x})$, which denotes the temperature distribution in Ω, satisfies the equation

$$k_{11}\frac{\partial^2 u(\boldsymbol{x})}{\partial x_1^2} + k_{22}\frac{\partial^2 u(\boldsymbol{x})}{\partial x_2^2} + k_{33}\frac{\partial^2 u(\boldsymbol{x})}{\partial x_3^2} + 2k_{12}\frac{\partial^2 u(\boldsymbol{x})}{\partial x_1 \partial x_2} + 2k_{13}\frac{\partial^2 u(\boldsymbol{x})}{\partial x_1 \partial x_3} + 2k_{23}\frac{\partial^2 u(\boldsymbol{x})}{\partial x_2 \partial x_3} = 0, \tag{1}$$