Available online at www.sciencedirect.com

ScienceDirect

Comput. Methods Appl. Mech. Engrg. 308 (2016) 535–551

Computer methods in applied mechanics and engineering

www.elsevier.com/locate/cma

Homogenization of coupled flow and deformation in a porous material

Carl Sandström*, Fredrik Larsson, Kenneth Runesson

Department of Applied Mechanics, Chalmers University of Technology, S-412 96 Gothenburg, Sweden

Received 2 August 2015; received in revised form 17 May 2016; accepted 20 May 2016
Available online 1 June 2016

Abstract

In this paper we present a framework for computational homogenization of the fluid–solid interaction that pertains to the coupled deformation and flow of pore fluid in a fluid-saturated porous material. Large deformations are considered and the resulting problem is established in the material setting. In order to ensure a proper FE-mesh in the fluid domain of the RVE, we introduce a fictitious elastic solid in the pores; however, the adopted variational setting ensures that the fictitious material does not obscure the motion of the (physical) solid skeleton. For the subsequent numerical evaluation of the RVE-response, hyperelastic properties are assigned to the solid material, whereas the fluid motion is modeled as incompressible Stokes' flow. Variationally consistent homogenization of the standard first order is adopted. The homogenization is selective in the sense that the resulting macroscale (upscaled) porous media model reminds about the classical one for a quasi-static problem with displacements and pore pressure as the unknown macroscale fields. Hence, the (relative) fluid velocity, i.e. seepage, "lives" only on the subscale and is part of the set of unknown fields in the RVE-problem. As to boundary conditions on the RVE, a mixture of Dirichlet and weakly periodic conditions is adopted. In the numerical examples, special attention is given to an evaluation of the Biot coefficient that occurs in classical phenomenological models for porous media.

MSC: 76S05; 74Q05; 74F10; 76D07

Keywords: Porous media; Homogenization; Fluid–structure interaction; Finite strains; Stokes' flow

1. Introduction

We consider the problem of fluid flow through deformable porous materials. Porous materials are present in a vast number of natural as well as engineered structures. Examples of natural structures include biological tissue and aquifers, while examples of engineered structures are foams and textiles. The microstructure of porous materials is generally very complex with characteristics at a lengthscale much smaller than the scale of the application; hence, it is computationally not feasible to solve the fully resolved problem. Consequently, macroscopic phenomenological

* Corresponding author.
E-mail addresses: carl.sandstrom@chalmers.se (C. Sandström), fredrik.larsson@chalmers.se (F. Larsson), kenneth.runesson@chalmers.se (K. Runesson).

http://dx.doi.org/10.1016/j.cma.2016.05.021

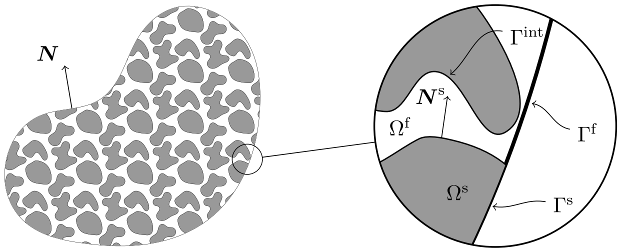

Fig. 1. Fully resolved domain in the material configuration with magnification of a small part of the boundary and pertinent symbols.

material models, based on a priori homogenization, are commonly used. Starting from Biot [1], a large number of so-called "porous media theories" of various complexity have been developed. An important class is the so-called Porous Media Theory (PMT) [2]. Although computationally efficient, PMT models are less capable of representing the intrinsic physical properties of the material.

As a viable alternative, computational homogenization [3] may be used, whereby the material response is evaluated using a Representative Volume Element [4] (RVE) that contains a small subset of the fully resolved microstructure. To ensure that the response from the RVE is physically feasible, boundary conditions satisfying the Hill–Mandel condition [5,6] must be chosen.

This paper concerns the homogenization of a fluid-filled (saturated) porous material. The pore system is assumed to be open, and we restrict the analysis to two phases; one solid and one fluid phase. Since we consider 3D microstructures, the solid skeleton is assumed to be contiguous in order to be able to sustain loads. In particular, we account for the interaction between the deformable solid and fluid, which represents a Fluid–Structure-Interaction (FSI) problem. The homogenization of flow in deformable porous media is also addressed by Iliev et al. [7] where the special case of flow through a deformable channel using asymptotic expansion.

We restrict to the case of laminar and incompressible flow of the pore fluid. For the FSI-problem, we use a monolithic approach with a conforming interface mesh. In order to maintain a proper mesh in the fluid domain during deformation, we introduce a fictitious elastic material in such a way that the mesh in the fluid domain follows the deformation of the solid. Measures are taken in order to ensure that the presence of the fictitious elastic material does not contribute to the overall stiffness of the RVE.

The paper is outlined as follows: In Section 2, the fine scale FSI-problem is established (in the material format). In Section 3, we introduce the homogenization scheme in the setting of the Variational Multiscale method and identify the macroscale problem. In Section 4, we discuss the RVE-problem. In Section 5, two numerical examples are presented. Finally, in Section 6 conclusions and future work are discussed.

2. Fine-scale fluid–structure interaction problem

2.1. Preliminaries

As a starting point, we consider the fully resolved domain in the material configuration depicted in Fig. 1, where the gray area represents the solid and the white area the fluid. We consider two phases, a solid phase, which is located in Ω^{s} and a fluid phase located in Ω^{f}. We also define the total domain and boundary as $\Omega = \Omega^{\mathrm{f}} \cup \Omega^{\mathrm{s}}$ and $\Gamma = \Gamma^{\mathrm{f}} \cup \Gamma^{\mathrm{s}}$. Γ^{f} is the part of Ω^{f} where fluid can enter and exit the domain and Γ^{s} is the part of Ω^{s} on the outer boundary. The interface between the solid and fluid phases is introduced as $\Gamma^{\mathrm{int}} = \partial\Omega^{\mathrm{s}} \cap \partial\Omega^{\mathrm{f}}$.

In order to allow for subsequent boundary conditions on both phases, we split Γ^{f} and Γ^{s} into a Dirichlet part and a Neumann part. Thus, for the solid boundary, we have $\Gamma^{\mathrm{s}} = \Gamma^{\mathrm{s}}_{\mathrm{u}} \cup \Gamma^{\mathrm{s}}_{\mathrm{t}}$ where the displacement is prescribed on $\Gamma^{\mathrm{s}}_{\mathrm{u}}$ (Dirichlet) and the traction is prescribed on $\Gamma^{\mathrm{s}}_{\mathrm{t}}$ (Neumann). Likewise, on the fluid part of the boundary, we have $\Gamma^{\mathrm{f}} = \Gamma^{\mathrm{f}}_{\mathrm{v}} \cup \Gamma^{\mathrm{f}}_{\mathrm{p}}$ where the velocity is prescribed on $\Gamma^{\mathrm{f}}_{\mathrm{v}}$ (Dirichlet) and the pressure on $\Gamma^{\mathrm{f}}_{\mathrm{p}}$ (Neumann). $\boldsymbol{N}$ is the outward pointing normal to Γ and $\boldsymbol{N}^{\mathrm{s}}$ is the normal on Γ^{int}, pointing outwards from Ω^{s}.

Note that, due to the forthcoming discretization of the domain and the interaction between the solid and the fluid, the computational mesh in Ω^{f} needs to follow the deformation of the solid in order to avoid the possibility of a distorted mesh. In a staggered approach to FSI problems, a mesh of poor quality in the fluid part of the domain should be updated after the solid deformation is computed in each iteration, using e.g. Laplace smoothing or by remeshing.