ARTICLE IN PRESS

Available online at www.sciencedirect.com

ScienceDirect

Comput. Methods Appl. Mech. Engrg. ▮ (▮▮▮▮) ▮▮▮–▮▮▮

Computer methods in applied mechanics and engineering

www.elsevier.com/locate/cma

Isogeometric Boundary Element analysis with elasto-plastic inclusions. Part 1: Plane problems

Gernot Beer[a,b,*], Benjamin Marussig[a], Jürgen Zechner[a], Christian Dünser[a], Thomas-Peter Fries[a]

[a] *Institute of Structural Analysis, Graz University of Technology, Lessingstraße 25/II, 8010 Graz, Austria*

[b] *Centre for Geotechnical and Materials Modelling, University of Newcastle, Callaghan, NSW 2308, Australia*

Abstract

In this work a novel approach is presented for the isogeometric Boundary Element analysis of domains that contain inclusions with different elastic properties than the ones used for computing the fundamental solutions. In addition the inclusion may exhibit inelastic material behavior. In this paper only plane stress/strain problems are considered.

In our approach the geometry of the inclusion is described using NURBS basis functions. The advantage over currently used methods is that no discretization into cells is required in order to evaluate the arising volume integrals. The other difference to current approaches is that Kernels of lower singularity are used in the domain term. The implementation is verified on simple finite and infinite domain examples with various boundary conditions. Finally a practical application in geomechanics is presented.

Keywords: BEM; Isogeometric analysis; Elasto-plasticity; Inclusions

1. Introduction

Isogeometric analysis [1] has gained significant popularity in the last decade because of the fact that geometry data can be taken directly from Computer Aided Design (CAD) programs, potentially eliminating the need for mesh generation. A true companion to CAD is the Boundary Element Method (BEM) because both employ a surface definition of the problem to be solved.

However, with a pure surface discretization the BEM can only analyze homogeneous, elastic domains. The method will be extended here to include heterogeneous, inelastic domains by introducing volume effects. We explain this on an elastic domain with an inclusion V_0 where body forces are present. Using the theorem of Betti as explained in [2],

* Corresponding author at: Institute of Structural Analysis, Graz University of Technology, Lessingstraße 25/II, 8010 Graz, Austria. Tel.: +43 316 873 6181; fax: +43 316 873 6185.

E-mail address: gernot.beer@tugraz.at (G. Beer).

URL: http://www.ifb.tugraz.at (G. Beer).

http://dx.doi.org/10.1016/j.cma.2016.03.035

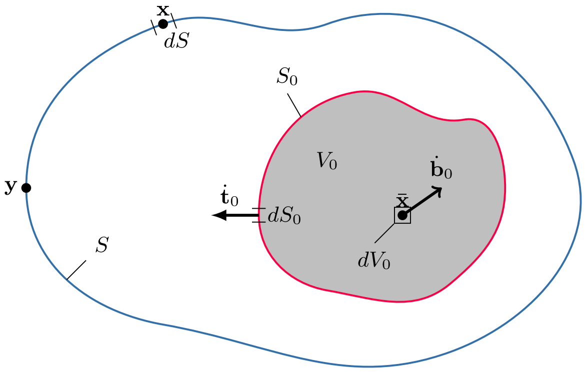

Fig. 1. Explanation of the derivation of the integral equation with volume effects.

the boundary integral equation can be written in incremental form and in matrix notation as:

$$\mathbf{c}\,\dot{\mathbf{u}}(\mathbf{y}) = \int_S \mathbf{U}(\mathbf{y},\mathbf{x})\dot{\mathbf{t}}(\mathbf{x})\,\mathrm{d}\,S + \int_{S_0} \mathbf{U}(\mathbf{y},\bar{\mathbf{x}})\dot{\mathbf{t}}_0(\bar{\mathbf{x}})\,\mathrm{d}\,S_0 - \int_S \mathbf{T}(\mathbf{y},\mathbf{x})\dot{\mathbf{u}}(\mathbf{x})\,\mathrm{d}\,S + \int_{V_0} \mathbf{U}(\mathbf{y},\bar{\mathbf{x}})\dot{\mathbf{b}}_0(\mathbf{x})\,\mathrm{d}\,V_0 \tag{1}$$

where $\mathbf{c}$ is a free term, $\mathbf{U}(\mathbf{y},\mathbf{x})$ and $\mathbf{T}(\mathbf{y},\mathbf{x})$ are matrices containing fundamental solutions for the displacements and tractions at a point $\mathbf{x}$ due to a source at a point $\mathbf{y}$ [3], $\dot{\mathbf{u}}(\mathbf{x})$ and $\dot{\mathbf{t}}(\mathbf{x})$ are increments of the displacement and traction vectors on the surface S, defining the problem domain (see Fig. 1). $\dot{\mathbf{b}}_0(\bar{\mathbf{x}})$ are increments of body force inside the inclusion and $\dot{\mathbf{t}}_0(\bar{\mathbf{x}})$ are increments of tractions related to the body force acting on surface S_0 bounding V_0.

Remark 1. In all previous work on elasto-plasticity, integral equations are used that involve a higher singularity Kernel in the volume integral and the direct use of initial stresses instead of body forces. Here we use a different approach involving body forces and a lower singularity Kernel. The derivation of the integral equations is shown in the Appendix.

The integral equations can be solved for the unknowns $\mathbf{u}$ or $\mathbf{t}$ by discretization. As in majority of previous work on the isogeometric BEM [4–10] we use the collocation method, i.e. we write the integral equations for a finite number N source points $\mathbf{y}_n$

$$\begin{aligned}\mathbf{c}\,\dot{\mathbf{u}}\left(\mathbf{y}_n\right) = &\int_S \mathbf{U}\left(\mathbf{y}_n,\mathbf{x}\right)\dot{\mathbf{t}}\left(\mathbf{x}\right)\mathrm{d}\,S + \int_{S_0} \mathbf{U}\left(\mathbf{y}_n,\bar{\mathbf{x}}\right)\dot{\mathbf{t}}_0\left(\bar{\mathbf{x}}\right)\mathrm{d}\,S_0 \\ &- \int_S \mathbf{T}\left(\mathbf{y}_n,\mathbf{x}\right)\dot{\mathbf{u}}\left(\mathbf{x}\right)\mathrm{d}\,S + \int_{V_0} \mathbf{U}\left(\mathbf{y}_n,\bar{\mathbf{x}}\right)\dot{\mathbf{b}}_0\left(\bar{\mathbf{x}}\right)\mathrm{d}\,V_0\end{aligned} \tag{2}$$

with $n = \{1,\ldots,N\}$. For the discretization of the surface integrals over S we divide the boundary into patches and use a geometry independent field approximation approach for each patch, i.e. we use different basis functions for the description of the geometry and for the field values.

$$\begin{aligned}\mathbf{x}^e &= \sum_{k=1}^{K} R_k(u)\,\mathbf{x}_k^e \\ \mathbf{u}^e &= \sum_{k=1}^{K^d} R_k^d(u)\,\mathbf{u}_k^e \\ \mathbf{t}^e &= \sum_{k=1}^{K^t} R_k^t(u)\,\mathbf{t}_k^e.\end{aligned} \tag{3}$$

In above equations the superscript e refers to the number of the patch, R_k, R_k^d and R_k^t are NURBS basis functions with respect to the local coordinate u of the parametrization for the geometry, displacements and tractions respectively.

Please cite this article in press as: G. Beer, et al., Isogeometric Boundary Element analysis with elasto-plastic inclusions. Part 1: Plane problems, Comput. Methods Appl. Mech. Engrg. (2016), http://dx.doi.org/10.1016/j.cma.2016.03.035