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A shear deformable, rotation-free isogeometric shell formulation

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Abstract

A finite element formulation for a geometrically linear, shear deformable (Reissner–Mindlin type) shell theory is presented, which exclusively uses displacement degrees of freedom. The total displacement is subdivided into a part representing the membrane and bending deformation, enriched by two extra "shear displacements", representing transverse shear deformation. This rotation-free approach is accomplished within the isogeometric concept, using C^1-continuous, quadratic NURBS as shape functions. The particular displacement parametrization decouples transverse shear from bending and thus the formulation is free from transverse shear locking by construction, i.e. locking is avoided on the theory level, not by choice of a particular discretization. Compared to the hierarchic formulation proposed earlier within the group of the authors (Echter et al., 2013), the method presented herein avoids artificial oscillations of the transverse shear forces. Up to now, a similar, displacement based method to avoid membrane locking has not been found. Thus, in the present formulation the mixed method from Echter et al. (2013) is used to avoid membrane locking.

Keywords: Isogeometric finite elements; Rotation-free; Shells; Hierarchic degrees of freedom; Locking; Displacement split

1. Introduction

The isogeometric approach has been proposed by Hughes and co-workers in 2005 [1,2]. In a nutshell, it is an isoparametric finite element method, using B-Splines, NURBS (non-uniform rational B-splines) or other CAD-typical parameterizations to generate shape functions. It thus provides a technology to directly link CAD and CAE in problems of structural mechanics and other fields.

In the present contribution, however, geometry approximation is not the focus. Instead, it is a different distinct feature of spline or NURBS discretizations, namely the fact that inter-element continuity can be easily controlled. More precisely, construction of C^1-continuous approximation spaces, being an awkward task in standard FEM, is quite easily done when using splines. This applies to shape functions within so-called patches. Continuity between patches, however, is an issue which requires extra attention.

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In fact, using splines for finite element analysis is not an entirely new idea (see, for instance Höllig [3] or Leung [4]), but somehow it never reached the level of maturity it has gained now in the context of the isogeometric concept.

The contribution by Kiendl et al. [5] was one of the first to exploit this distinctive feature to formulate an isogeometric discretization of Love's classical shell theory (Kirchhoff [6], Love [7]). The underlying weak form has a variational index of 2 and thus requires C^1-continuous shape functions. Traditionally, this has been a major obstacle for development of corresponding finite elements and it is one of the reasons for the popularity of shear deformable finite elements, in spite of their pertinent locking problems. An alternative approach to NURBS is the use of subdivision surfaces. Corresponding thin shell elements have been proposed by Çirak et al. [8], Çirak and Ortiz [9]. This technology, however, is not further pursued in the present study.

Isogeometric shear deformable (Reissner–Mindlin type) plate and shell elements have been proposed by Adam et al. [10], Beirão Da Veiga et al. [11], Benson et al. [12], [13], Caseiro et al. [14], Dornisch et al. [15] and Hosseini et al. [16]. In these contributions locking is either ignored, using high order polynomials to obtain acceptable accuracy, or it is alleviated by adoption of known concepts, like reduced integration and assumed strain formulations.

In contrast to Kirchhoff–Love type thin shells, C^0-continuous shape functions are sufficient for consistency of Reissner–Mindlin shell elements. Nevertheless, higher continuity discretization may have outstanding merits. Echter et al. [17] (see also [18]) have presented on this basis a hierarchic family of isogeometric shell finite elements. It has some remarkable features:

- Kirchhoff–Love shells, Reissner–Mindlin shells and a three-dimensional shell formulation including thickness stretch are combined in one unique, hierarchic framework.
- Owing to the hierarchic concept, the different shell models can be activated or de-activated by simply switching on or off the corresponding degrees of freedom. Different types of shell theories can be mixed within one patch if this is required, e.g. in the context of model adaptivity.
- Due to the particular parametrization of the kinematic equations, the formulation avoids transverse shear locking (see also [19]) and curvature thickness locking (trapezoidal locking), the latter being relevant for a three-dimensional shell model with thickness change. No particular shape functions or variational formulations are needed. Beirão da Veiga et al. [20] present a mathematical analysis of this approach and exploit the aforementioned feature by applying it to a plate formulation using NURBS and NURPS (non-uniform rational Powell–Sabin splines).

Shear deformable shell finite elements with distinct degrees of freedom for transverse shear have been described earlier by Oñate and Zárate [21] as well as Zárate and Oñate [22]. Due to the separate treatment of transverse shear they are free from transverse shear locking and contain the thin shell limit (i.e. Kirchhoff–Love theory) as a special case. In contrast to the work by Echter et al. [17], however, these elements rely on a mixed variational formulation, based on the Hu–Washizu functional. Three-dimensional shell finite elements with an exact Kirchhoff–Love limit, also based on a mixed formulation, have been presented by Bischoff and Taylor [23].

The essential contribution of the present paper consists in a modification and extension of the original concept from Echter et al. [17], which provides significant advantages. Instead of hierarchic rotations, only displacement degrees of freedom are used. To the authors' best knowledge, this is the first time that a rotation-free version of shell finite elements with Reissner–Mindlin kinematics, based on a primal (purely displacement based) formulation is described, without resorting to a mixed variational principle.

As a result, certain stress oscillations are removed and an optimal balance of the involved function spaces is obtained a priori. The new method is a straightforward evolution of the original idea from [17], promoting it to a new canonical concept in formulation of isogeometric finite elements for thin-walled structures. Presentation of the proposed discretization concept is restricted to geometrically linear problems so far. Extension to non-linear problems is work in progress.

In order to further elaborate on the advantages of the present formulation it is necessary to reiterate in more detail the nature of locking phenomena in shells and their particular significance for isogeometric finite element formulations. This is done in the following section.

2. Locking in the isogeometric context

It is a well known fact that purely displacement based standard finite element formulations for shells suffer from locking problems, resulting in severe underestimation of deformations, dependency of the error on shell thickness