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Topology optimization of structures with interval random parameters

Ning Chen[a], Dejie Yu[a,*], Baizhan Xia[a], Zhengdong Ma[b]

[a] *State Key Laboratory of Advanced Design and Manufacturing for Vehicle Body, Hunan University, Changsha, Hunan, 410082, People's Republic of China*
[b] *Department of Mechanical Engineering, University of Michigan, Ann Arbor, MI 48105, USA*

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Abstract

The aim of this paper is to present a robust topology optimization methodology for structures with imprecise probability uncertainty. In this paper, the imprecise probability uncertainties are treated with an interval random model, in which the probability variables are used to model the uncertain parameters and some distribution parameters of probability variables are expressed as interval variables instead of a precise value. In the presented methodology, the deterministic topology optimization techniques and the hybrid stochastic interval perturbation method (HSIPM) are combined to obtain robust topology designs for structures with interval random parameters. The exploitation of HSIPM transforms the problem of topology optimization with interval random parameters into an augmented deterministic topology optimization problem. This provides a computationally cheap alternative to Monte Carlo-based optimization algorithms. Several numerical examples are presented to demonstrate the effectiveness of the proposed method.

Keywords: Topology optimization; Random distribution; Interval variable; Random moment method; Perturbation method

1. Introduction

Topology optimization is one of the most general forms of structural optimization [1–3]. It seeks the optimal layout of the material within the design domain to optimize the structural connectivity and material distribution simultaneously. Some algorithms have been put forward for structural topology optimization. These algorithms are usually performed with the assumption that the system parameters are deterministic, which means that the uncertainties that exist in either the fabrication processes or the operating conditions are ignored. However, solutions obtained by performing the optimization in a deterministic setting may be impractical or suboptimal when considering the inherent uncertainties in real-world engineering conditions. Because these uncertainties may cause significant variability in structural performance, they must be properly considered in the optimal structural design problem. There are a variety of approaches to design optimization under uncertainty [4,5]. In general, two types of structural optimization formulations have been developed to handle system optimization with uncertainties: reliability-based

* Corresponding author. Tel.: +86 731 88821915; fax: +86 731 88823946.
E-mail address: djyu@hnu.edu.cn (D. Yu).

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design optimization (RBDO) and robust design optimization (RDO). There are conceptual differences between these two formulations. The former often addresses optimal design problems considering the probability of failure and optimizes performance metrics subject to a reliability index [6,7], whereas the latter emphasizes finding a design to minimize the influence of uncertain variability on the response and allows for the maximum possible system variability in everyday service conditions [8,9].

Many studies have been devoted to reliability-based topology optimization (RBTO) in different settings and under different sources of uncertainty [10–13]. In this paper, we consider robust design and thus limit the discussion to robust topology optimization (RTO). Most of the existing formulations of RTO are based on probabilistic description of the uncertain variations. Specifically, the bi-criteria robust design optimization formulation has been adopted, in which both the nominal value and the variance of the goal performance function are to be minimized. Several researchers have proposed RTO algorithms to consider uncertainty in applied loads and structural stiffness. Lógó et al. [14,15] presented RTO algorithms using a first-order approximation for compliance in the presence of uncertainty in applied loads. De Gournay et al. [16] used a level set approach to minimize the worst-case compliance in shape and topology optimization under the perturbation of loading. Conti et al. [17] formulated a level-set-based shape optimization method under stochastic loading through a two-stage stochastic programming approach. Guest et al. [18] studied topology optimization of structures with load and nodal position uncertainties described by joint probability density functions. Asadpoure et al. [19] carried out robust topology optimization of truss structures with stiffness uncertainties by transforming the non-deterministic topology optimization problem into an augmented deterministic one. Recently, Chen et al. [20,21] studied robust shape and topology optimization under random loading, material properties and geometrical uncertainties. Lazarov1 et al. [22] presented a topology optimization methodology for obtaining robust designs insensitive to small uncertainties in the geometry.

As mentioned above, probabilistic RTO approaches have been well established. The common theme of these RTO approaches is that they seek an alternate deterministic formulation by manipulating the uncertainties. Nevertheless, these probabilistic RTO approaches generally rely on precise data on the probability distribution of the random parameters, such as expectation and standard variance. However, in some real engineering applications, these data are not always available and are sometimes very difficult to obtain due to a lack of sufficient samples. In such situations, we must make some suitable assumptions for probability distributions of random variables. However, these assumed probability distributions may be unreliable, and based on them, the results obtained by the probabilistic approach may be incorrect [23]. Therefore, research on probabilistic RTO with imprecise probability distribution data is in demand and quite promising. For this reason, the interval random model is introduced herein to describe the uncertainties of parameters involved in topology optimization without sufficient information to construct precise probability distributions. In the interval random model, the uncertain parameters are treated as random variables whose distribution parameters with limited information can be given only as interval variables instead of precise values. This uncertainty model was first proposed to find the least favourable value of the mean-square response of a random vibration problem by Elishakoff and Colombi [24,25]. Subsequently, the interval random model has gained much attention in the literature when dealing with imprecise probability uncertainties. It has been applied to the structural response analysis [26] and the structural reliability analysis [27–31].

In this paper, the interval random model is employed to handle the uncertainty involved in the topology optimization process, which has not been exploited to date. We first present a hybrid stochastic interval perturbation method (HSIPM) for the static analysis of the structure with interval random parameters, which will be deduced concretely in Section 3. A key benefit of the proposed approach is that it can reduce a significant amount of computational burden when quantifying the behaviour of the uncertain structure compared with the Monte Carlo method. Using HSIPM, the formulation of the RTO with interval random parameters is then studied. The problem of topology optimization with interval random parameters is transformed to an augmented deterministic topology optimization problem with the help of HSIPM. Therein, the maximum values of the expectation and the standard variance of the structural compliance are to be minimized. In the numerical examples, the cases in which the interval random uncertainty involved in topology optimization is treated as corresponding interval uncertainty or random uncertainty are also investigated. By comparing these robust optimal designs, the result of topology optimization with interval random uncertainty performs the best. This indicates that the interval random uncertainty in topology optimization cannot be substituted by interval uncertainty or random uncertainty.

The remainder of this paper is organized as follows. In Section 2, the deterministic topology optimization is introduced. In Section 3, a hybrid stochastic interval perturbation method for the static analysis of the structure with