Available online at www.sciencedirect.com
ScienceDirect
Comput. Methods Appl. Mech. Engrg. 306 (2016) 1–18
Computer methods in applied mechanics and engineering
www.elsevier.com/locate/cma

Maximum-entropy methods for time-harmonic acoustics

F. Greco*, L. Coox, W. Desmet

KU Leuven, Department of Mechanical Engineering, Division PMA, Celestijnenlaan 300B - box 2420, B-3001 Leuven, Belgium

Received 22 October 2015; received in revised form 26 February 2016; accepted 28 March 2016
Available online 5 April 2016

Highlights

- Time harmonic acoustic problems are simulated with maximum entropy meshless methods.
- Maximum entropy basis functions handle dispersion errors better than finite elements.
- Short wavelength propagation problems can be studied with coarser discretizations.
- A blending with Isogeometric functions is possible on the boundary of the domain.
- Several examples are studied, including a 2D car cavity model defined by B-Splines.

Abstract

This paper explores the application of *maximum-entropy* methods (max-ent) to time harmonic acoustic problems. Max-ent basis functions are mesh-free approximants that are constructed observing an equivalence between basis functions and discrete probability distributions and applying Jaynes's *maximum entropy* principle. They are C^{∞}-continuous and therefore they are particularly suited for the resolution of Helmholtz problems, where classical finite element methods show a poor accuracy in the high frequency region. In addition, it was recently shown that max-ent approximants can be blended with isogeometric basis functions on the boundary of the domain. This preserves the correct representation of the boundary like in Isogeometric Analysis, with the advantage that the discretization of the interior of the domain is straightforward. In this paper the max-ent mathematical formulation is reviewed and then some numerical applications are studied, including a 2D car cavity geometry defined by B-spline curves. In all cases, if the same nodal discretization is used, finite elements results are significantly improved.

Keywords: Maximum-entropy; Meshless; High geometric fidelity; Helmholtz problems

1. Introduction

Over the last few years the acoustic properties of a product have become an important criterion in many design problems. This increasing demand on the acoustic performance raised a strong need for numerical prediction tools which allow a reliable evaluation of different design alternatives without expensive experimental studies.

* Corresponding author.
E-mail address: francesco.greco@kuleuven.be (F. Greco).

http://dx.doi.org/10.1016/j.cma.2016.03.047
0045-7825/

Several techniques are nowadays available for the simulation of the acoustic wave propagation, addressed by the Helmholtz equation. Among these, the Finite Element Method (FEM) [1] and the Boundary Element Method (BEM) [2] have been widely studied in the literature. The most important advantage of BEM is the dimension reduction of the physical problem. However, it requires significant computational times because it works with full, complex and frequency-dependent matrices. For this reason, the FEM is the most employed numerical technique in commercial simulation tools for interior problems. Nevertheless, some drawbacks are still present. It is well known in the FEM literature that the accuracy of the numerical results heavily depends on the regularity of the mesh used for the discretization [3]. Although many powerful mesh generation algorithms are available nowadays [4], this operation may take a significant part of the total analysis time, especially in three-dimensional applications. In addition, the mesh generation process can rarely be automated and therefore important human resources are required in the preprocessing stage. Another well known problem that affects the FEM is the poor accuracy in the high frequency region, due to pollution errors, [1,5,6] which makes the short wave acoustic problem one of the still unsolved problems in the FEM environment [7].

Numerical methods with higher continuity of the basis functions, such as Isogeometric Analysis (IGA) [8], are expected to better handle the latter drawback. In [9] IGA is applied to the simulation of structural vibration problems and it is shown to outperform the standard FEM in the high frequency region. The main purpose of IGA is to integrate the Computer Aided Design (CAD) and the analysis stages by using the same basis functions for the geometric representation and for the numerical analysis. By doing so, the geometrical errors that are introduced by the FEM discretization on the boundary of the domain are avoided. Unfortunately IGA does not possess the same flexibility as the standard FEM in discretizing complexly shaped domains. Some modified formulations based on T-splines [10], hierarchical B-splines [11] and trimming techniques [12] have been proposed for two-dimensional applications but the problem is still open in 3D. In [13] an isogeometric boundary element method based on T-splines is proposed; with this approach the interior parametrization of the domain is no longer required, but as in the classical BEM, fully populated matrices are obtained. In contrast to this method, which is a direct BEM, the NURBS-based technique proposed in [14] is an indirect BEM, which also allows the modeling of open boundary domains. Such boundaries are very common in vibroacoustic problems.

Another family of higher order continuity schemes, whose application to acoustic problems has been recently studied, are the mesh-free (or meshless) approximation schemes [15]. The earliest meshless methods are the somehow equivalent Element Free Galerkin Method (EFGM) [16] and Reproducing Kernel Particle Method (RKPM) [17]. These methods use a Moving Least Squares approach [18] to construct the basis functions that, as a consequent drawback, are not strictly positive and do not possess the Kronecker-Delta property on the boundary of the domain, which requires additional efforts to impose essential boundary conditions [19]. The latter problem is solved in the Point Interpolation Method (PIM) [20] where polynomial interpolants that pass through each node are obtained. By using Radial Basis Functions, the RPIM, which is better suited for arbitrarily scattered sets of points, was subsequently developed [21].

An interpolatory approximation is also obtained in the Natural Element Method (NEM) [22], where the basis functions are constructed using the Delaunay triangulation and the Voronoi diagram of the nodes. NEM basis functions are non-negative and possess the Kronecker-Delta property, but their evaluation requires a significant computational effort [23].

In more recent studies, the Jaynes's *maximum entropy* principle [24] was used for the construction of polygonal interpolants [25] and then generalized for the definition of the *local maximum-entropy* (LME) mesh-free approximants [26]. In contrast to the previous mesh-free schemes these approximants are C^{∞} everywhere within the convex-hull of the computational grid, they are strictly non-negative and they possess the weak Kronecker-Delta property on the faces of the convex-hull. Additionally, it was recently shown that the max-ent formalism can be used to blend the LME approximants with isogeometric basis functions [27]. In particular, the boundary of the domain is described with a NURBS curve and isogeometric functions are associated to the control points that define the curve. Then the LME approximants are calculated in the interior of the domain and they are blended with the NURBS functions on a thin region close to the boundary. Thanks to this approach, the IGA difficulties in the parametrization of complexly shaped surfaces are avoided and, at the same time, the geometrical error on the boundary, which like for FEM is present also for Galerkin based meshless formulations, can be avoided as well.

Despite their recent introduction, many applications of max-ent methods have emerged over the last years, including thin shell analysis [28], reduced order modeling of mechanical systems [29], convection–diffusion