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Gradient based design optimization under uncertainty via stochastic expansion methods

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Abstract

We present a computational framework for robust and reliability based design optimization which combines stochastic expansion methods, namely polynomial chaos expansion, with design sensitivity analysis. It is well known that the statistical moments and their gradients with respect to design variables can be readily obtained from the polynomial chaos expansion. However, the evaluation of the failure probabilities of the cost and constraint functions and their gradients, requires integrations over failure regions. To simplify this we introduce an indicator function into the integrand, whereby the integration region becomes the known range of random variables and to alleviate the non-differentiable property of the indicator function, a smooth approximation is adopted to facilitate the sensitivity analysis. Both intrusive and non-intrusive polynomial chaos approaches for uncertainty propagation are employed in the design optimization of linear elastic structures. Guidelines to assess the computational costs associated with both polynomial chaos approaches are also presented.

Keywords: Reliability based design optimization; Robust design optimization; Polynomial chaos expansion; Gradient based optimization

1. Introduction

Deterministic optimization has been extensively used for the design of engineering systems, wherein the inherent uncertainty is accounted for by safety factors or worst-case design scenarios. These approaches result in designs that may be either too conservative or unknowingly insufficient. This deficiency can be overcome by employing uncertainty analysis in the design optimization process as done in the robust and reliability based design approaches. The robust approach produces designs that are relatively insensitive to variations in the system parameters [1–4], while the reliability based approach produces designs that satisfy probabilistic performance criteria [5–9].

There are two main approaches for robust design optimization. In both methods the cost function is penalized by its variance. The first approach calculates the robustness measures (the statistical moments) in an analytical manner using numerical techniques such as the Taylor series expansion. The second approach calculates the robustness measures

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through a Monte Carlo strategy of given designs. In both cases nonlinear programming algorithms are employed for the design optimization [10–13].

Reliability based design optimization ensures failure probabilities with respect to random loads, etc. are sufficiently small. Numerical approximation techniques are used to estimate the failure probability such as the First-Order Reliability Methods (FORM) and Second-Order Reliability Methods (SORM) [14,15]. Both approaches approximate the limit state function, i.e. the failure function via a Taylor series expansion. As their names suggest, the former uses a first-order expansion while the latter uses a second-order expansion. Errors are encountered in these probability estimates if the limit state function is neither linear (for the FORM) or quadratic (for the SORM) and if the random variables are not Gaussian. Incorporating these methods in structural optimization leads to nested optimization problems which are difficult to solve.

In this paper a computational framework adopting stochastic expansion methods is integrated into structural optimization [16,17]. Namely, we use the polynomial chaos (PC) expansion to obtain accurate and efficient response approximations for systems with uncertain parameters. PC methods provide a functional representation of the random response with respect to its random input which can be used to estimate statistical moments of optimization cost and constraint functions. Moreover sensitivities i.e. gradients of these functions with respect to the optimization design variables are readily obtained [18–21]. Other response quantities, namely failure probabilities and their gradients are more difficult to compute because they require integration over failure regions, i.e. subsets of the random space which are not explicitly known. This problem is addressed in [22] where the authors employ stochastic expansions in conjunction with FORM for the reliability assessment and sensitivity analysis. While their approach enjoys the efficiency of the stochastic expansion methods, it suffers from the previously mentioned issues associated with FORM. To compute failure probabilities we introduce an indicator function to transform the failure region to the full random space. The non-differentiable property of the indicator function is mitigated by adopting a smooth approximation to facilitate the sensitivity analysis. Direct differentiation and adjoint sensitivity analysis formulations are derived incorporating both intrusive and non-intrusive PC expansions. These approaches are elucidated through both simple spring validation studies and truss optimization problems.

The paper is organized as follows. Section 2 introduces the uncertainty analysis approaches. Section 3 presents the design sensitivity analysis in the presence of uncertainty. Design optimization problems involving a linear elastic truss are solved in Section 4 using the various approaches and estimates of their computational costs are provided. Finally, Section 5 concludes this paper.

2. Uncertainty propagation

In this section we summarize the uncertainty analysis method via PC expansion. Let an experiment that quantifies the response of a system be described by the probability triple $(\Omega, \mathcal{F}, P)$, where Ω is the set of all possible outcomes, $\mathcal{F}$ is the set of events and P is a function that maps the events to the probabilities. Real-valued random variables X relevant to this experiment are measurable functions $X : \Omega \mapsto \mathbb{R}$. We restrict our attention to independent second-order random variables, i.e. independent random variables with finite variance (applicable to most physical processes) that can be expressed as a convergent series in terms of polynomials [23]

$$X(\omega) = \hat{x}_0 H_0 + \sum_{i_1=1}^{\infty} \hat{x}_{i_1} H_1(\xi_{i_1}(\omega)) + \sum_{i_1=1}^{\infty} \sum_{i_2=1}^{i_1} \hat{x}_{i_1 i_2} H_2(\xi_{i_1}(\omega), \xi_{i_2}(\omega)) + \sum_{i_1=1}^{\infty} \sum_{i_2=1}^{i_1} \sum_{i_3=1}^{i_2} \hat{x}_{i_1 i_2 i_3} H_3(\xi_{i_1}(\omega), \xi_{i_2}(\omega), \xi_{i_3}(\omega)) + \cdots, \quad (1)$$

where the polynomials $H_n(\xi_{i_1}, \ldots, \xi_{i_n})$ are functions of the random variables $\boldsymbol{\xi} = (\xi_{i_1}, \ldots, \xi_{i_n})$ at the realization $\omega \in \Omega$. The equation above is re-written in a notationally convenient form as

$$X(\omega) = \sum_{i=0}^{\infty} x_i \psi_i(\boldsymbol{\xi}(\omega)), \quad (2)$$

where there is a one-to-one correspondence between the polynomials $\psi_i(.)$ and $H_n(.)$. For notational simplicity, henceforth the dependence on ω is seen through the random variables $\boldsymbol{\xi}$. Accordingly, the above equation is written