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Multi-Index Stochastic Collocation for random PDEs

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Abstract

In this work we introduce the Multi-Index Stochastic Collocation method (MISC) for computing statistics of the solution of a PDE with random data. MISC is a combination technique based on mixed differences of spatial approximations and quadratures over the space of random data. We propose an optimization procedure to select the most effective mixed differences to include in the MISC estimator: such optimization is a crucial step and allows us to build a method that, provided with sufficient solution regularity, is potentially more effective than other multi-level collocation methods already available in literature. We then provide a complexity analysis that assumes decay rates of product type for such mixed differences, showing that in the optimal case the convergence rate of MISC is only dictated by the convergence of the deterministic solver applied to a one dimensional problem. We show the effectiveness of MISC with some computational tests, comparing it with other related methods available in the literature, such as the Multi-Index and Multilevel Monte Carlo, Multilevel Stochastic Collocation, Quasi Optimal Stochastic Collocation and Sparse Composite Collocation methods.

Keywords: Uncertainty Quantification, Random PDEs, Multivariate approximation, Sparse grids, Stochastic Collocation methods, Multilevel methods, Combination technique.
2010 MSC: 41A10, 65C20, 65N30, 65N05

1. Introduction

Uncertainty Quantification (UQ) is an interdisciplinary, fast-growing research area that focuses on devising mathematical techniques to tackle problems in engineering and natural sciences in which only a probabilistic description of the parameters of the governing equations is available, due to measurement errors, intrinsic non-measurability/non-predictability, or incomplete knowledge of the system of interest. In this context, "parameters" is a term used in broad sense to refer to constitutive laws, forcing terms, domain shapes, boundary and initial conditions, etc.

UQ methods can be divided into deterministic and randomized methods. While randomized techniques, which include the Monte Carlo sampling method, are essentially based on random sampling and ensemble averaging, deterministic methods proceed by building a surrogate of the system's response function over the parameter space, which is then processed to obtain the desired information. Typical goals include computing statistical moments (expected value, variance, higher moments, correlations) of some quantity of interest of the system at hand, typically functionals of the state variables (forward problem), or updating the statistical description of the random parameters given some observations of the system at hand (inverse problem). In any case, multiple resolutions of the governing equations are needed to explore the dependence of the state variables on the random parameters. The computational method used should therefore be carefully designed to minimize the computational effort.

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