Available online at www.sciencedirect.com
ScienceDirect
Comput. Methods Appl. Mech. Engrg. 306 (2016) 196–215
Computer methods in applied mechanics and engineering
www.elsevier.com/locate/cma

Data-driven operator inference for nonintrusive projection-based model reduction

Benjamin Peherstorfer*, Karen Willcox

Department of Aeronautics & Astronautics, MIT, 77 Massachusetts Avenue, Cambridge, MA 02139, USA

Received 7 September 2015; received in revised form 2 February 2016
Available online 6 April 2016

Abstract

This work presents a nonintrusive projection-based model reduction approach for full models based on time-dependent partial differential equations. Projection-based model reduction constructs the operators of a reduced model by projecting the equations of the full model onto a reduced space. Traditionally, this projection is intrusive, which means that the full-model operators are required either explicitly in an assembled form or implicitly through a routine that returns the action of the operators on a given vector; however, in many situations the full model is given as a black box that computes trajectories of the full-model states and outputs for given initial conditions and inputs, but does not provide the full-model operators. Our nonintrusive operator inference approach infers approximations of the reduced operators from the initial conditions, inputs, trajectories of the states, and outputs of the full model, without requiring the full-model operators. Our operator inference is applicable to full models that are linear in the state or have a low-order polynomial nonlinear term. The inferred operators are the solution of a least-squares problem and converge, with sufficient state trajectory data, in the Frobenius norm to the reduced operators that would be obtained via an intrusive projection of the full-model operators. Our numerical results demonstrate operator inference on a linear climate model and on a tubular reactor model with a polynomial nonlinear term of third order.

Keywords: Nonintrusive model reduction; Data-driven model reduction; Black-box full model; Inference

1. Introduction

Model reduction seeks to construct reduced models that provide accurate approximations of the full model solutions with orders of magnitude reduction in computational complexity. We consider here projection-based model reduction for full models that are based on parametrized time-dependent partial differential equations (PDEs). Projection-based model reduction first constructs a basis of a low-dimensional reduced space and then projects the equations of the full model onto the reduced space to obtain the operators of the reduced model [1–5]. The construction of these reduced operators is usually intrusive and requires the full-model operators, which means that the full-model operators need to be available either in an assembled form or through a routine that returns the action of the operators on a given

* Corresponding author. Tel.: +1 617 253 7831.
E-mail address: pehersto@mit.edu (B. Peherstorfer).

http://dx.doi.org/10.1016/j.cma.2016.03.025
0045-7825/

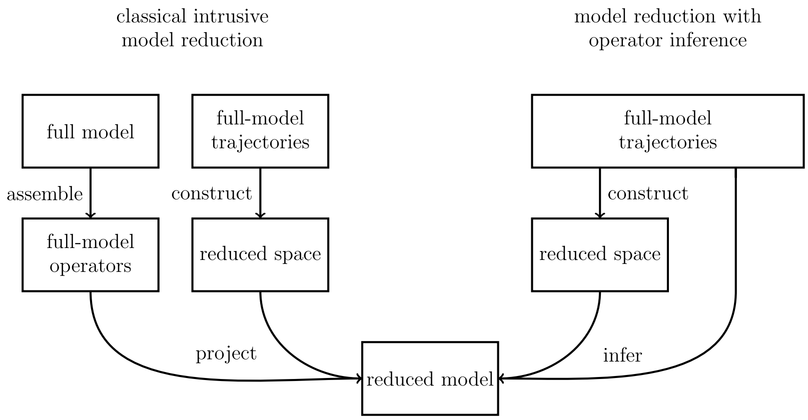

Fig. 1. The projection step in classical projection-based model reduction is intrusive because the operators of the full model are required for the construction of the reduced operators. In contrast, our operator inference derives the approximations of the reduced operators directly from data (initial conditions, inputs, trajectories, outputs) of the full model. This operator inference is nonintrusive and therefore applicable to black-box full models.

vector. This limits the scope of projection-based model reduction because in many situations the full model is given as a black box, which allows computing the trajectories of the full-model states and outputs for given initial conditions and inputs, but does not provide the full-model operators. Note that we also consider full models as black-box models if the source code of the implementation is available but the complexity of the code renders obtaining the full-model operators time consuming, see the general circulation model (GCM) [6] in Section 4.4. We present nonintrusive operator inference that replaces the classical intrusive reduced operator construction by deriving approximations of the reduced operators directly from data of the full model, without requiring the full-model operators. The data include initial conditions, inputs, trajectories of the full-model states, and outputs. The reduced operators are low-dimensional quantities and therefore the inference underlying our approach is feasible with respect to computational costs and with respect to required amount of data. Our operator inference is applicable to nonlinear PDEs with polynomial nonlinear terms of low order. Our operator inference provides a nonintrusive way to construct a reduced model, see Fig. 1.

There are several classical projection-based model reduction methods. Proper orthogonal decomposition (POD) constructs a reduced basis from states of the full model [1,2,7]. The reduced basis method [3,8] derives a reduced basis with a greedy approach [9] based on computationally cheap *a posteriori* error estimators of intermediate reduced models. There are also Krylov subspace methods, including multivariate Padé approximations and tangential interpolation [4,5,10]. Once the reduced basis is generated, all these methods construct a reduced model and the reduced operators with an intrusive projection step, leading to intrusive model reduction methods. In [11], the coefficients of a reduced state for a given input are approximately derived from a computationally cheap coarse-grid full-model state. This avoids the construction of reduced operators and leads to a nonintrusive approximate reduced basis method; however, the computation of the coefficients requires solving the equations of the coarse-grid full model at each input for which the reduced system of the reduced model is solved.

The Loewner framework provides a nonintrusive approach to construct reduced models of linear time-invariant (LTI) systems. The reduced operators are derived directly from frequency response data, i.e., from transfer function evaluations [12–14], without requiring the operators of the full LTI system. The Loewner framework has been extended to parametric LTI systems [15] and to LTI systems with multiple outputs [16]; however, compared to intrusive projection-based model reduction, the scope of the Loewner framework is still limited. In particular, an extension of the Loewner framework to nonlinear models remains an open problem. There is also vector fitting [17,18] that constructs approximate rational interpolants of LTI systems for given frequency response data. Both the Loewner framework and vector fitting rely on frequency response data, which are often unavailable for full models that are formulated in the time domain and marched forward in time with a time stepping scheme.

In contrast to projection-based reduced models, data-fit surrogate models are constructed using interpolation and regression techniques to directly learn the map from inputs to outputs of the full model [19–22]. The construction of a reduced model and its operators is therefore circumvented and with it the intrusive projection step. The approaches in [23–30] construct a reduced space and learn a surrogate model that maps inputs to coefficients of the representa-