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A Fractional Phase-Field Model for Two-Phase Flows with Tunable Sharpness: Algorithms and Simulations ⋆

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Highlights

- A new fractional mass-conserving Allen-Cahn model.
- A second-order (time) spectral (space) method for the coupled system of fractional equations.
- A variable-fractional order model to control multi-rate diffusion.
- First numerical solution of the fractional Navier-Stokes equations.

Abstract

We develop a fractional extension of a mass-conserving Allen-Cahn phase field model that describes the mixture of two incompressible fluids. The fractional order controls the sharpness of the interface, which is typically diffusive in integer-order phase-field models. The model is derived based on an energy variational formulation. An additional constraint is employed to make the Allen-Cahn formulation mass-conserving and comparable to the Cahn-Hilliard formulation but at reduced cost. The spatial discretization is based on a Petrov-Galerkin spectral method whereas the temporal discretization is based on a stabilized ADI scheme both for the phase-field equation and for the Navier-Stokes equation. We demonstrate the spectral accuracy of the method with fabricated smooth solutions and also the ability to control the interface thickness between two fluids with different viscosity and density in simulations of two-phase flow in a pipe and of a rising bubble. We also demonstrate that using a formulation with variable fractional order we can deal simultaneously with both erroneous boundary effects and sharpening of the interface at no extra resolution.

Key words: Fractional Allen-Cahn equation; Sharp interface; Eulerian method; ADI; Spectral method; Variable fractional order